Operations with Polynomials

Algebra I

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Big Picture

Monomials and polynomials can contain numbers, variables, and exponents. They can be added, subtracted, multiplied, divided, and factored, just like real numbers. There are a few special products of polynomials that are important to know, such as the product of two binomials.

Key Terms

Monomial: A number, a variable with a positive integer exponent, or the product of a number and variable(s) with positive integer exponents.

Polynomial: A monomial or sum of monomials.

Term: A part of the polynomial that is added or subtracted.

Coefficient: A number that appears in front of a variable.

Constant: A number without a variable.

Binomial: A polynomial with two terms.

Trinomial: A polynomial with three terms.

Standard Form: A form where the terms in the polynomial are arranged in order of decreasing power (exponents decrease from left to right).

Leading Coefficient: The coefficient of the term with the greatest power.

Degree of a Monomial: Sum of the exponents in the monomial.

Degree of a Polynomial: The greatest degree of the terms.

Like Terms: Terms in the polynomial with the same exponents (coefficients could be different).

Terminology

Examples of **monomials**:

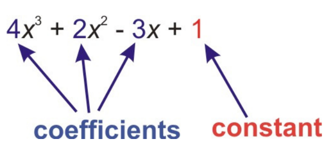
- $7, \frac{1}{2}x, 3a^2b$

These are not monomials:

- $\frac{3}{x}, 2^a, x^{-1}$

A **polynomial** is made up of different **terms** that contain positive integer powers of the variables.

- A term can be a **coefficient** with a variable or just a **constant**.
- A polynomial with only two terms is called **binomial**, and a polynomial with only three terms is called a **trinomial**.
- If the terms are written in **standard form** so that the exponents decreased from left to right, the first coefficient is the **leading coefficient.**



- 4 is the coefficient of x^3 and is the leading coefficient

Degrees:

- $4x^3$ has degree 3
- $2x^2$ has degree 2
- $-3x$ has degree 1
- 1 has degree 0
- The **degree of the polynomial** is 3.

Addition & Subtraction of Polynomials

- To add 2 or more polynomials, write their sum and combine **like terms**. Once there are no more like terms, the polynomial is simplified.
- To subtract 1 polynomial from the other, add the opposite of each term of the polynomial we are subtracting.

Example: $(4x^2-3xy+2) + (2x^3+5y) - (x^2+5xy-3)$

Group like terms: $(2x^3) + (4x^2-x^2) + (-3xy-5xy) + (5y) + (2-(-3))$

Simplify: $2x^3 + 3x^2 - 8xy + 5y + 5$

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Operations with Polynomials cont.

Multiplication of Polynomials

Multiplying Monomials

- Multiply the coefficients as we would any number and use the product rule for exponents.
- The product rule for exponents is $x^n \cdot x^m = x^{n+m}$.

Multiplying Polynomials

- Use the distributive property so that every term in one polynomial is multiplied by every other term in the other polynomial.
- The distributive property is $a(b+c) = ab+ac$.

Another method is called **FOIL**. If given $(a+b)(c+d)$:

- Multiply the **F**irst terms in each polynomial (a, c)
- Multiply the **O**utermost terms in each polynomial (a, d)
- Multiply the **I**nnermost terms in each polynomial (b, c)
- Multiply the **L**ast terms in each polynomial (b, d)
- Combine any like terms

So $(a+b)(c+d) = ac + ad + bc + bd$

Polynomials can be multiplied vertically, similar to vertical multiplication with regular numbers.

Example: (a+b)(c+d)

$$\begin{array}{rr} & a+b \\ \times & c+d \\ \hline & ad+bd \\ +\ ac+bc & \\ \hline & ac+ad+bc+bd \end{array}$$

Division of Polynomials

Dividing Monomials

- Write as a fraction and use the quotient of powers.
- The quotient rule for exponents is $\frac{x^n}{x^m} = x^{n-m}$.

Dividing Polynomials

- To divide a polynomial by a monomial, we can divide each term in the numerator by the monomial.
  - Example: $\frac{3x^3+6x-1}{x} = \frac{3x^3}{x} + \frac{6x}{x} - \frac{1}{x} = 3x^2 + 6 - \frac{1}{x}$
- To divide a polynomial by a binomial, use long division.
  - $\text{Dividend} \div \text{Divisor} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$
  - The dividend is the numerator, and the divisor is the denominator.

For example: $\frac{x^2+4x+5}{x+3}$

```
          x+1
x+3 ) x² + 4x +5
      -x²  - 3x
      -----------
            x + 5
           -x - 3
           -------
              2
```

So $\frac{x^2+4x+5}{x+3} = x + 1 + \frac{2}{x+3}$

Tips:

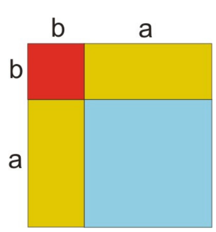
- Rewrite the polynomial in standard form.
- Write any missing terms with zero coefficients.
  - Example: Rewrite $2x^2+3$ as $2x^2+0x+3$

Special Products of Polynomials

Square of a Binomial

$(a+b)^2 = (a+b)(a+b)$
$= a^2+ab+ab+b^2$
$= a^2+2ab+b^2$

You can also remember the square of the binomial by drawing this diagram:



The area of the square is $(a+b)(a+b) = (a+b)^2$

- The area can be found by adding up the four smaller squares and rectangles.
- $(a+b)(a+b) = a^2+2ab+b^2$

We can also find $(a-b)^2$ by replacing b with $-b$:

$(a-b)^2 = (a-b)(a-b)$
$= a^2-ab-ab-b^2$
$= a^2-2ab+b^2$

Sum and Difference Patterns

$(a+b)(a-b) = a^2+ab-ab+b^2$
$= a^2-b^2$

a and b can represent numbers, variables, or variable expressions.