Properties of Exponents

Big Picture

When an exponent is a positive whole number, an exponent is a shorthand way to represent large quantities of multiplication. Exponents can represent any value that is multiplied by itself repeatedly (for example, an expression such as 5·5·5·5). Variables can have exponents just like real numbers. When the exponent is zero, negative, or fractional, there are certain rules we have to remember.

Key Terms

Power: An expression with a base and an exponent.

Exponent: A number or symbol written as the superscript to the upper right of another number that is a type of mathematical operation.

Exponential Form

For the **power** x^n,

- x is the base
- n is the **exponent** (or the power)

We don't usually write out the exponent if $n = 1$, so $x^1 = x$. If $n = 2$, we say x squared. If $n = 3$, we say x cubed.

If n is a positive whole number, an exponent is a short-hand notation for repeated multiplication.

- Example: $x^5 = x \cdot x \cdot x \cdot x \cdot x$
- Example: $(3a)^4 = (3a)(3a)(3a)(3a)$, which can be simplified: $3 \cdot 3 \cdot 3 \cdot 3 \cdot a \cdot a \cdot a \cdot a = 81a^4$

Exponents of negative numbers:

- Even powers of negative numbers are positive

$$(-2)^6 = (-2)(-2)(-2)(-2)(-2)(-2) = \underbrace{(-2)(-2)}_{+4} \cdot \underbrace{(-2)(-2)}_{+4} \cdot \underbrace{(-2)(-2)}_{+4} = +64$$

- Odd powers of negative numbers are negative

$$(-2)^5 = (-2)(-2)(-2)(-2)(-2) = \underbrace{(-2)(-2)}_{+4} \cdot \underbrace{(-2)(-2)}_{+4} \cdot \underbrace{(-2)}_{-2} = -32$$

Properties Involving Products

Product Rule for Exponents

To multiply powers with the same base, add the exponents together.

- $x^m \cdot x^n = x^{m+n}$

For example, $x^5 \cdot x^3$ can be written out as:

$$\underbrace{(x \cdot x \cdot x \cdot x \cdot x)}_{x^5} \cdot \underbrace{(x \cdot x \cdot x)}_{x^3} = \underbrace{(x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x)}_{x^8}$$

Don't multiply the bases! $2^2 \cdot 2^3 = 2^5$, NOT 4^5.

Power Rule for Exponents

To take a power of a product, multiply the exponents together.

- $(x^m)^n \cdot x^{mn}$
- $(xy)^m \cdot x^m y^m$

Properties Involving Quotients

Quotient Rule for Exponents

To divide powers with the same base, subtract the exponents.

- $\frac{x^m}{x^n} = x^{m-n}$

Power Rule for Quotients

To take a power of a quotient, multiply that exponent to the exponent of the numerator and the exponent of the denominator.

- $\left(\frac{x^m}{x^n}\right)^p = \frac{x^{mp}}{x^{np}}$

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Properties of Exponents

Zero, Negative, Fractional Exponents

Zero Rule for Exponents

Any number raised to the zero power is equal to 1.

- $x^0 = 1$

If you can't remember this, use the quotient rule where the exponents of the numerator and the denominator are equal.

$$\frac{x^n}{x^n} = x^{(n-n)} = x^0 = 1$$

Negative Power Rule for Exponents

A number raised to a negative number is the same as a fraction with the exponent in the denominator.

- $x^{-n} = \frac{1}{x^n}$ and $x \neq 0$

Example: $2^{-2} = \frac{1}{2^2}$

Negative Power Rule for Fractions

A fraction raised to a negative number is the same as taking the reciprocal of the fraction, then raising the numerator and denominator to the same exponent.

- $\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^{n} = \left(\frac{y^n}{x^n}\right)$, where $x \neq 0$ and $y \neq 0$

Rule for Fractional Exponents

A number raised to a fractional number is the same as taking the root of it.

- $x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$

Notes