Representing Functions

Big Picture

Functions are a way of relating one variable to another variable in order to find solutions for both variables. Graphs are a great way to visually represent functions and can often help the solution finding process.

Key Terms

Coordinate Plane: Used to represent sets of numbers. Has two axes and an origin.

Origin: The point where two axes intersect.

Quadrant: Any one of four sections of the coordinate plane created by the two axes.

Coordinate: A pair of numbers that describe how far along the *x*-axis and *y*-axis a point is.

Function: A relationship between two variables (an input and an output) where there is exactly one output for each input.

Independent Variable: The input variable - a variable whose value does not depend on the value of another variable.

Dependent Variable: The output variable - a variable whose value depends on the value of an input variable.

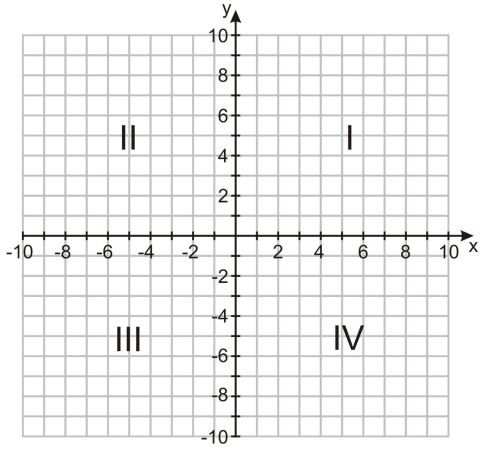
Domain: The set of values for the input variable of a function.

Range: The set of values for the output variable of a function.

Coordinate Plane

The **coordinate plane** can be used to represent pairs of numbers.

- Looks like two number lines that meet at right angles. These "number lines" are called axes (singular, axis).
  - The horizontal line is called the *x*-axis. The positive *x*-direction points to the right.
  - The vertical line is called the *y*-axis. The positive *y*-direction points upward.
- The **origin** is where the two axes intersect
- The axes split the coordinate plane into 4 **quadrants**
  - Moving counter-clockwise starting from the upper right quadrant, the quadrants are numbered I, II, III, and IV



Coordinates

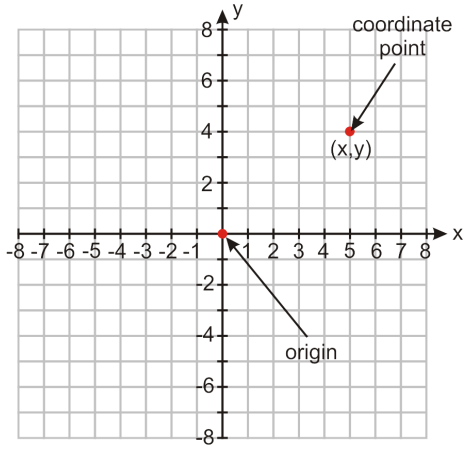
A point on the coordinate plane is determined by its **coordinates** (*x, y*).

- The order that the coordinates are written in matters! Coordinates are always written with the *x*-coordinate first.
- The coordinates of the origin are (0, 0).

The x-coordinate is also called the abscissa. The y-coordinate is also called the ordinate.

In the graph below, the coordinates of the point are (5, 4).

- To reach the point from the origin, move 5 units to the *right* and 4 units *upward*.
- A negative *x*-coordinate means moving to the *left*, and a negative *y*-coordinate means moving *downward*.



Notes

This guide was created by Nicole Crawford, Jane Li, and Jin Yu. To learn more about the student authors, visit http://www.ck12.org/about/ck-12-interns/.

Representing Functions cont.

Functions

Graphs are useful for showing relations and functions.

- A relation relates two or more variables (such as x and y).
  - An equation with two variables is an example of a relation. The two variables are related by performing the operations of the equation (multiplication, addition, etc.).
- A **function** is a type of relationship that relates one variable called an input to another variable called an output. The relation is such that there is *exactly* 1 output for every input.
- Typically x represents the input variable and y represents the output variable.
  - The input variable (x) is also called the **independent variable** because its value is not controlled by another variable.
  - The output variable (x) is the **dependent variable** because the output *depends* on the input variable.
- A function is often written as f(x), which stands for *function* of x.
  - Examples: f(x) = 2x-5, f(x) = x^3,
- Sometimes f(x) is replaced with y.

Domain and Range

The **domain** and **range** describe the values that would make the function true.

- The domain is all the possible x-values (input values) of a function.
  - If the domain is all real numbers, the function is *continuous*. If the domain is a particular set of numbers (such as whole numbers only), the function is discrete.
- The range is all the possible y-values (output values) of a function.
  - The range of the function depends on the domain.

To find the domain of a function, determine which input values would make sense. Usually this means the input should produce a real number as the output. For example, if an x-value would make the denominator equal to 0, the output wouldn't be a real number.

- Example: $f(x) = \frac{1}{x-2}$ Domain: $[x \neq 2]$

  This means all real numbers *except* $x = 2$ are a part of the domain. If $x = 2$, the denominator would equal 0.

The easiest way to find the range of a function is to look at the graph and find where on the y-axis there are no x-values.

- Example: f(x) = x^2 Range: $[y \geq 0]$

  This means all real numbers *greater than or equal* to 0 are a part of the range. x^2 is always positive, so it is not possible for the y-value to be less than 0.

Graphing Functions

A function can be graphed once the relation is known. The relation could be a rule or a table of values.

Example: Graph the function that has the following table of values.

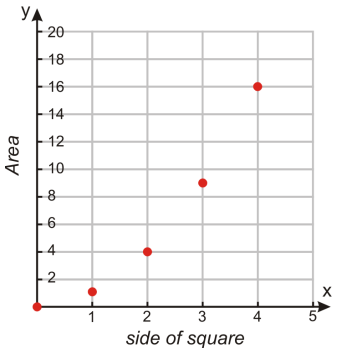
Side of a square	0	1	2	3	4
Area of a square	0	1	4	9	16

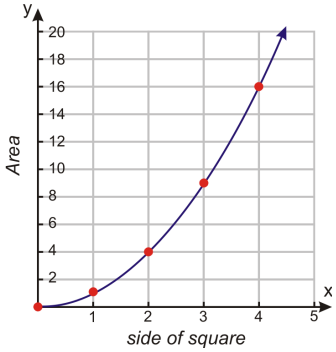
The graphs (to the right) only show quadrant I. It does not make sense to have a negative side or negative area, so they are not included in the domain and range.

When graphing functions, make sure to label the axes.

If drawing a graph when given a rule instead of a table of values, make a table of some values to plot.

- Usually picking integer values is best because it makes calculations easier.
- Connect the points if the graph is continuous.





Vertical Line Test

When graphing a relation, the vertical line test can be used to see if the relation is a function.

- If you can draw a vertical line that crosses the graph in more than one place, the relation is *not* a function.