Direct Variation

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CHAPTER 1 Direct Variation

Here you'll determine if a set of data is related directly.

According to Newton's second law, the net force (F) of an object is equal to its mass (m) times its acceleration (a), where F is measured in Newtons, m is measured in kilograms, and a is measured in meters/sec/sec. If an object with an acceleration of 8 meters/sec/sec has a force of 24 Newtons, what is the object's force when its acceleration is 12 meters/sec/sec?

Guidance

We say that a set of data is related **directly** if the independent and dependent variables both grow large or small together. For example, the equation of the line $y = 2x$ would represent a direct variation relationship. As x gets bigger, so would y. In fact, direct variation equation is $y = kx, k \neq 0$, which looks just like the equation of a line without a y-intercept. We call k the **constant of variation** and y is said to vary directly with x. k can also be written $k = \frac{y}{x}$.

Example A

The variables x and y vary directly, and $y = 10$ when $x = 2$. Write an equation that relates x and y and find y when $x = 9$.

Example B

Determine if the set of data varies directly. If so, find the direct variation equation.

TABLE 1.1:

x	4	8	16	20
y	1	2	4	5

Example C

The number of calories, C, a person burns working out varies directly with length of time it was done, t (in minutes). A 150 pound person can burn 207 calories swimming laps for 30 minutes. Write a variation model for C as a function of t. Then, determine how long it will take that person to burn 520 calories.

Intro Problem Revisit If we write Newton's second law as an equation, we get $F = ma$. We can now see that this is an example of a direct variation equation, where $y = F$, $m = k$, and $a = x$. Using the direct variation equation, we can substitute in F and a and solve for m.

$$F = ma$$
$$24 = m(8)$$
$$3 = m$$

So the mass of the object is 3 kg but we're looking for its force when its acceleration is 12 meters/sec/sec. Hence, we use the formula again.

$$F = ma$$
$$F = (3)(12)$$
$$F = 36$$

Therefore, the object's force is 36 Newtons.

Guided Practice

1. x and y vary directly. When $x = -8, y = -6$. Find the equation and determine x when $y = 12$.

2. Determine if the set below varies directly.

TABLE 1.2:

x	1	2	3	4	5
y	2	4	8	16	20

3. Taylor's income varies directly with the number of hours he works. If he worked 60 hours last week and made $900, how much does he make per hour? Set up a direct variation equation.

Practice

For problems 1-4, use the given x and y values to write a direct variation equation and find y given that $x = 12$.

1. $x = 3, y = 15$
2. $x = 9, y = -3$
3. $x = \frac{1}{2}, y = \frac{1}{3}$
4. $x = -8, y = \frac{4}{3}$

For problems 5-8, use the given x and y values to write a direct variation equation and find x given that $y = 2$.

5. $x = 5, y = 4$
6. $x = 18, y = 3$
7. $x = 7, y = -28$
8. $x = \frac{2}{3}, y = \frac{5}{6}$

Determine if the following data sets vary directly.

9.

TABLE 1.3:

x	12	16	5	20
y	3	4	1	5

10.

TABLE 1.4:

x	2	10	5	6
y	14	70	35	42

11.

TABLE 1.5:

x	2	8	18	34
y	3	12	27	51

Solve the following word problems using a direct variation equation.

12. Based on her weight and pace, Kate burns 586 calories when she runs 5 miles. How many calories will she burn if she runs only 3 miles? How many miles (to the nearest mile) does she need to run each week if she wants to burn one pound (3500 calories) of body fat each week?
13. One a road trip, Mark and Bill cover 450 miles in 8 hours, including stops. If they maintain the same pace, how far (to the nearest mile) will they be from their starting point after 15 hours of driving?
14. About three hours into a fundraising car wash, the Mathletes Club earned $240 washing 48 cars. How much was charged for each carwash? How many more cars will they have to wash to reach their goal of earning $400?
15. Dorothy earned $900 last week for working 36 hours. What is her hourly wage? If she works full time (40 hours) in a week how much will she make?