Graphing Linear Equations

Big Picture

Graphs and functions are critical, not only for solving math problems, but for real life situations. They can be used, for example, to find trends in data. None of this is possible, however, without first knowing the basic foundation of graphing, the different forms that an equation can be written in, or how to write these equations. These basics will be used for all types of more complex graphing in the future.

Key Terms

Slope: A ratio of the distance moved vertically over the distance moved horizontally in a non-vertical line.

***x*-Intercept:** The point where a line crosses the *x*-axis.

***y*-Intercept:** The point where a line crosses the *y*-axis.

Slope-Intercept Form: A form of writing a linear equation in two variables: $y = mx+b$, where m is the slope, b is the y-intercept, and x and y are the variables.

Slope

An equation in two variables can be graphed on a coordinate plane.

The graph is the set of points that are solutions to the equation (they make the equation true).

If the graph is a straight line, the equation is linear.

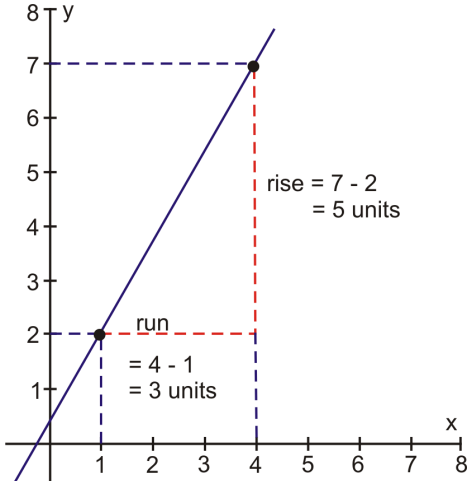
- A non-vertical line has a **slope**.

$$slope = m = \frac{rise}{run}$$

The slope between the points (x_1, y_1) and (x_2, y_2) is:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

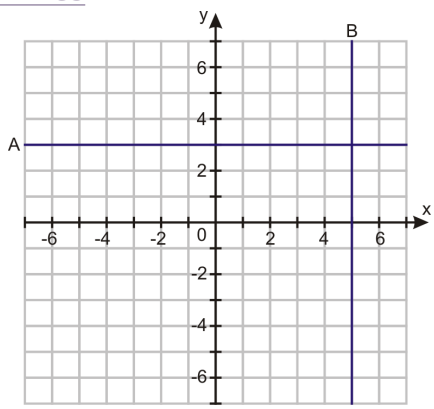
- Δ is the Greek letter delta that means *change*



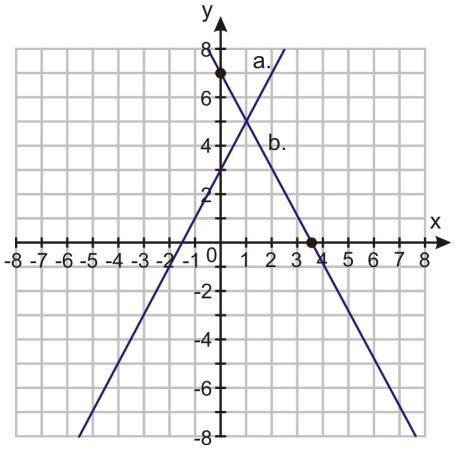
The slope m of this graph is 5/3.

Think of the slope as describing the steepness of the line. The slope can also represent a rate of change when one quantity is compared to another. Example: miles per hour

Slopes of Lines



- Horizontal line (line A in the graph above): $m = 0$
  - There is no rise, so the line is horizontal
  - $y = a$ is a horizontal line that passes through the point $(0, a)$
- Vertical line ($x = 5$, line B in the graph above): m is undefined
  - Dividing by 0 is undefined, so the slope of a vertical line is undefined
  - $x = b$ is a vertical line that passes through the point $(b, 0)$



- Positive slope (line a in the graph above): $m > 0$
  - The line goes up as we move to the right
- Negative slope (line b in the graph above): $m < 0$
  - The line goes down as we move to the right

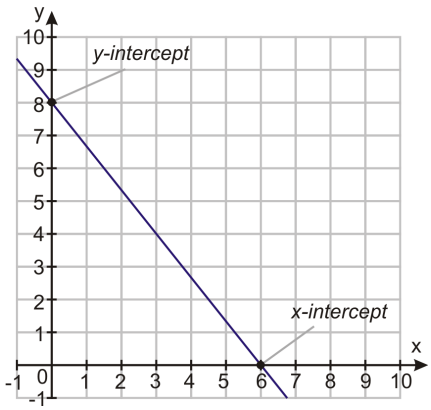
This guide was created by Nicole Crawford, Jane Li, and Jin Yu. To learn more about the student authors, visit http://www.ck12.org/about/ck-12-interns/.

Graphing Linear Equations cont.

Intercept

Two points can be used to determine a line. Two convenient points to use are the ***x*-intercept** and the ***y*-intercept**.

- The x-intercept is where $y = 0$
- The y-intercept is where $x = 0$

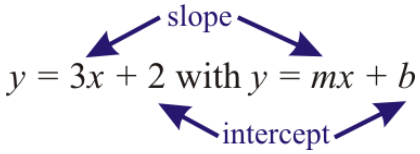


Linear Equations

Slope-Intercept Form

The **slope-intercept form** is a common form of writing a linear equation: $y = mx+b$

- x and y are the variables
- m is the slope
- b is the y-intercept, meaning the line goes through the point $(0, b)$



- If $b = 0$, y varies directly with x, and m is the constant of proportionality. x and y show a direct variation. If $b \neq 0$, the equation is not a direct variation.

To graph an equation in the slope-intercept form,

1. Draw $(0, b)$
2. Find another point by plugging in a value for x
3. Connect the two points

Other Forms

- Standard form: $Ax+By = C$ (this B is not the same as b in the slope-intercept form)
- Point-slope form: $y-y_0 = m(x-x_0)$, where (x_0, y_0) is a point on the line

Converting from standard to slope-intercept form:

$$Ax + By = C \rightarrow y = -\frac{A}{B}x + \frac{C}{B}$$

Converting from slope-intercept to point-slope form:

- The goal of point-slope form is to get x and y on opposite sides and have only integers as coefficients.
- Isolate x and y. If any of the coefficients are fractions, multiply the entire equation by the least common denominator of all the fractions.

Converting from point-slope to slope-intercept form:

- Distribute m and then isolate y by moving y_0 to the other side of the equation
  - $y-y_0 = m(x-x_0) \rightarrow y-y_0 = mx-mx_0 \rightarrow y = mx-mx_0 + y_0$
  - The slope = m, and the y-intercept = $-mx_0 + y_0$

Notes