CHAPTER 2

Chapter 2: Equations and Inequalities

Chapter Outline

2.1 Patterns and Equations

Introduction

In mathematics, and especially in algebra, we look for patterns in the numbers we see. The tools of algebra help us describe these patterns with words and with **equations** (formulas or functions). An equation is a mathematical recipe that gives the value of one variable in terms of another.

For example, if a theme park charges $12 admission, then the number of people who enter the park every day and the amount of money taken in by the ticket office are related mathematically, and we can write a rule to find the amount of money taken in by the ticket office.

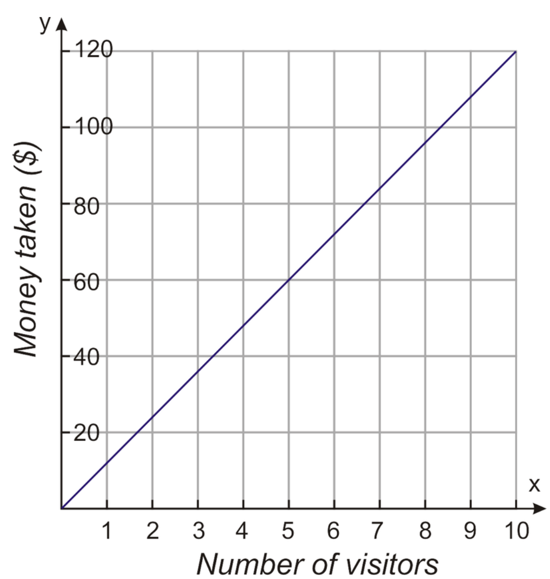
In words, we might say "The amount of money taken in is equal to twelve times the number of people who enter the park."

We could also make a table. The following table relates the number of people who visit the park and the total money taken in by the ticket office.

Number of visitors	1	2	3	4	5	6	7
Money taken in ($)	12	24	36	48	60	72	84

Clearly, we would need a **big** table to cope with a busy day in the middle of a school vacation!

A third way we might relate the two quantities (visitors and money) is with a graph. If we plot the money taken in on the **vertical axis** and the number of visitors on the **horizontal axis**, then we would have a graph that looks like the one shown below. Note that this graph shows a smooth line that includes non-whole number values of x (e.g. $x = 2.5$). In real life this would not make sense, because fractions of people can't visit a park. This is an issue of domain and range, something we will talk about later.



The method we will examine in detail in this lesson is closer to the first way we chose to describe the relationship. In words we said that *"The amount of money taken in is twelve times the number of people who enter the park."* In mathematical terms we can describe this sort of relationship with **variables**. A variable is a letter used to represent an unknown quantity. We can see the beginning of a mathematical formula in the words:

The amount of money taken in **is** twelve **times** the number of people who enter the park.

This can be translated to:

$$\text{the amount of money taken in} = 12 \times (\text{the number of people who enter the park})$$

We can now see which quantities can be assigned to **letters**. First we must state which letters (or **variables**) relate to which quantities. We call this **defining the variables**:

Let $x =$ the number of people who enter the theme park.

Let $y =$ the total amount of money taken in at the ticket office.

We now have a fourth way to describe the relationship: with an algebraic equation.

$$y = 12x$$

Writing a mathematical equation using variables is very convenient. You can perform all of the operations necessary to solve this problem without having to write out the known and unknown quantities over and over again. At the end of the problem, you just need to remember which quantities x and y represent.

Write an Equation

An equation is a term used to describe a collection of **numbers** and **variables** related through mathematical **operators**. An **algebraic equation** will contain letters that represent real quantities. For example, if we wanted to use the algebraic equation in the example above to find the money taken in for a certain number of visitors, we would substitute that number for x and then solve the resulting equation for y.

Example 1

A theme park charges $12 entry to visitors. Find the money taken in if 1296 people visit the park.

Let's break the solution to this problem down into steps. This will be a useful strategy for all the problems in this lesson.

Step 1: Extract the important information.

$$(\text{number of dollars taken in}) = 12 \times (\text{number of visitors})$$
$$(\text{number of visitors}) = 1296$$

Step 2: Translate into a mathematical equation. To do this, we pick variables to stand for the numbers.

$$\text{Let } y = (\text{number of dollars taken in}).$$
$$\text{Let } x = (\text{number of visitors}).$$

$$(\text{number of dollars taken in}) = 12 \times (\text{number of visitors})$$
$$y = 12 \times x$$

Step 3: Substitute in any known values for the variables.

$$\begin{aligned} y &= 12 \times x \\ x &= 1296 \\ &\therefore \\ y &= 12 \times 1296 \end{aligned}$$

Step 4: Solve the equation.

$$y = 12 \times 1296 = 15552$$

The amount of money taken in is $15552.

Step 5: Check the result.

If $15552 is taken at the ticket office and tickets are $12, then we can divide the total amount of money collected by the price per individual ticket.

$$(\text{number of people}) = \frac{15552}{12} = 1296$$

1296 is indeed the number of people who entered the park. **The answer checks out.**

Example 2

The following table shows the relationship between two quantities. First, write an equation that describes the relationship. Then, find out the value of b when a is 750.

a	0	10	20	30	40	50
b	20	40	60	80	100	120

Step 1: Extract the important information.

We can see from the table that every time a increases by 10, b increases by 20. However, b is not simply twice the value of a. We can see that when $a = 0,\ b = 20$, and this gives a clue as to what rule the pattern follows. The rule linking a and b is:

"To find b, double the value of a and add 20."

Step 2: Translate into a mathematical equation:

TABLE 2.1:

Text	Translates to	Mathematical Expression
"To find b"	$\rightarrow$	$b =$
"double the value of a"	$\rightarrow$	$2a$
"add 20"	$\rightarrow$	$+ 20$

Our equation is $b = 2a + 20$.

Step 3: Solve the equation.

The original problem asks for the value of b when a is 750. When a is 750, $b = 2a + 20$ becomes $b = 2(750) + 20$. Following the order of operations, we get:

$$\begin{aligned} b &= 2(750) + 20 \\ &= 1500 + 20 \\ &= 1520 \end{aligned}$$

Step 4: Check the result.

In some cases you can check the result by plugging it back into the original equation. Other times you must simply double-check your math. In either case, checking your answer is *always* a good idea. In this case, we can plug our answer for b into the equation, along with the value for a, and see what comes out. $1520 = 2(750) + 20$ is TRUE because both sides of the equation are equal. A true statement means that **the answer checks out**.

Use a Verbal Model to Write an Equation

In the last example we developed a **rule**, written in words, as a way to develop an algebraic **equation**. We will develop this further in the next few examples.

Example 3

The following table shows the values of two related quantities. Write an equation that describes the relationship mathematically.

TABLE 2.2:

$x-$**value**	$y-$**value**
-2	10
0	0
2	-10
4	-20
6	-30

Step 1: Extract the important information.

We can see from the table that y is five times bigger than x. The value for y is negative when x is positive, and it is positive when x is negative. Here is the rule that links x and y:

"y is the negative of five times the value of x"

Step 2: Translate this statement into a mathematical equation.

TABLE 2.3:

Text	**Translates to**	**Mathematical Expression**
"y is"	$\rightarrow$	$y =$
"negative 5 times the value of x"	$\rightarrow$	$-5x$

Our equation is $y = -5x$.

Step 3: There is nothing in this problem to **solve** for. We can move to Step 4.

Step 4: Check the result.

In this case, the way we would check our answer is to use the equation to generate our own *xy* pairs. If they match the values in the table, then we know our equation is correct. We will plug in -2, 0, 2, 4, and 6 for *x* and solve for *y*:

TABLE 2.4:

x	y
-2	$-5(-2) = 10$
0	$-5(0) = 0$
2	$-5(2) = -10$
4	$-5(4) = -20$
6	$-5(6) = -30$

The $y-$values in this table match the ones in the earlier table. **The answer checks out**.

Example 4

Zarina has a $100 gift card, and she has been spending money on the card in small regular amounts. She checks the balance on the card weekly and records it in the following table.

TABLE 2.5:

Week Number	**Balance ($)**
1	100
2	78
3	56
4	34

Write an equation for the money remaining on the card in any given week.

Step 1: Extract the important information.

The balance remaining on the card is not just a constant multiple of the week number; 100 is 100 times 1, but 78 is not 100 times 2. But there is still a pattern: the balance decreases by 22 whenever the week number increases by 1. This suggests that the balance is somehow related to the amount "-22 times the week number."

In fact, the balance equals "-22 times the week number, plus *something*." To determine what that *something* is, we can look at the values in one row on the table—for example, the first row, where we have a balance of $100 for week number 1.

Step 2: Translate into a mathematical equation.

First, we define our variables. Let n stand for the week number and b for the balance.

Then we can translate our verbal expression as follows:

TABLE 2.6:

Text	**Translates to**	**Mathematical Expression**
Balance equals -22 times the week number, plus *something*.	$\rightarrow$	$b = -22n + ?$

To find out what that ? represents, we can plug in the values from that first row of the table, where $b = 100$ and $n = 1$. This gives us $100 = -22(1) + ?$.

So what number gives 100 when you add -22 to it? The answer is 122, so that is the number the ? stands for. Now our final equation is:

$$b = -22n + 122$$

Step 3: All we were asked to **find** was the expression. We weren't asked to solve it, so we can move to Step 4.

Step 4: Check the result.

To check that this equation is correct, we see if it really reproduces the data in the table. To do that we plug in values for n:

$$\begin{aligned}
n = 1 \rightarrow b &= -22(1) + 122 = 122 - 22 = 100 \\
n = 2 \rightarrow b &= -22(2) + 122 = 122 - 44 = 78 \\
n = 3 \rightarrow b &= -22(3) + 122 = 122 - 66 = 56 \\
n = 4 \rightarrow b &= -22(4) + 122 = 122 - 88 = 34
\end{aligned}$$

The equation perfectly reproduces the data in the table. **The answer checks out**.

Solve Problems Using Equations

Let's solve the following real-world problem by using the given information to write a mathematical equation that can be solved for a solution.

Example 5

A group of students are in a room. After 25 students leave, it is found that $\frac{2}{3}$ *of the original group is left in the room. How many students were in the room at the start?*

Step 1: Extract the important information

We know that 25 students leave the room.

We know that $\frac{2}{3}$ of the original number of students are left in the room.

We need to find how many students were in the room at the start.

Step 2: Translate into a mathematical equation. Initially we have an unknown number of students in the room. We can refer to this as the original number.

Let's define the variable $x =$ the original number of students in the room. After 25 students leave the room, the number of students in the room is $x - 25$. We also know that the number of students left is $\frac{2}{3}$ of x. So we have two expressions for the number of students left, and those two expressions are equal because they represent the same number. That means our equation is:

$$\frac{2}{3}x = x - 25$$

Step 3: Solve the equation.

Add 25 to both sides.

$$\begin{aligned}
x - 25 &= \frac{2}{3}x \\
x - 25 + 25 &= \frac{2}{3}x + 25 \\
x &= \frac{2}{3}x + 25
\end{aligned}$$

Subtract $\frac{2}{3}x$ from both sides.

$$x - \frac{2}{3}x = \frac{2}{3}x - \frac{2}{3}x + 25$$
$$\frac{1}{3}x = 25$$

Multiply both sides by 3.

$$3 \cdot \frac{1}{3}x = 3 \cdot 25$$
$$x = 75$$

Remember that x represents the original number of students in the room. So, there were 75 students in the room to start with.

Step 4: Check the answer:

If we start with 75 students in the room and 25 of them leave, then there are $75 - 25 = 50$ students left in the room.

$\frac{2}{3}$ of the original number is $\frac{2}{3} \cdot 75 = 50$.

This means that the number of students who are left over equals $\frac{2}{3}$ of the original number. **The answer checks out**.

The method of defining variables and writing a mathematical equation is the method you will use the most in an algebra course. This method is often used together with other techniques such as making a table of values, creating a graph, drawing a diagram and looking for a pattern.

Review Questions

TABLE 2.7:

Day	Profit
1	20
2	40
3	60
4	80
5	100

1. The above table depicts the profit in dollars taken in by a store each day.
   a. Write a mathematical equation that describes the relationship between the variables in the table.
   b. What is the profit on day 10?
   c. If the profit on a certain day is $200, what is the profit on the next day?

   (a) Write a mathematical equation that describes the situation: *A full cookie jar has 24 cookies. How many cookies are left in the jar after you have eaten some?*
   (b) How many cookies are in the jar after you have eaten 9 cookies?
   (c) How many cookies are in the jar after you have eaten 9 cookies and then eaten 3 more?

2. Write a mathematical equation for the following situations and solve.
   a. Seven times a number is 35. What is the number?
   b. Three times a number, plus 15, is 24. What is the number?

c. Twice a number is three less than five times another number. Three times the second number is 15. What are the numbers?
d. One number is 25 more than 2 times another number. If each number were multiplied by five, their sum would be 350. What are the numbers?
e. The sum of two consecutive integers is 35. What are the numbers?
f. Peter is three times as old as he was six years ago. How old is Peter?

3. How much water should be added to one liter of pure alcohol to make a mixture of 25% alcohol?
4. A mixture of 50% alcohol and 50% water has 4 liters of water added to it. It is now 25% alcohol. What was the total volume of the original mixture?
5. In Crystal's silverware drawer there are twice as many spoons as forks. If Crystal adds nine forks to the drawer, there will be twice as many forks as spoons. How many forks and how many spoons are in the drawer right now?

(a) Mia drove to Javier's house at 40 miles per hour. Javier's house is 20 miles away. Mia arrived at Javier's house at 2:00 pm. What time did she leave?
(b) Mia left Javier's house at 6:00 pm to drive home. This time she drove 25% faster. What time did she arrive home?
(c) The next day, Mia took the expressway to Javier's house. This route was 24 miles long, but she was able to drive at 60 miles per hour. How long did the trip take?
(d) When Mia took the same route back, traffic on the expressway was 20% slower. How long did the return trip take?

6. The price of an mp3 player decreased by 20% from last year to this year. This year the price of the player is $120. What was the price last year?
7. SmartCo sells deluxe widgets for $60 each, which includes the cost of manufacture plus a 20% markup. What does it cost SmartCo to manufacture each widget?
8. Jae just took a math test with 20 questions, each worth an equal number of points. The test is worth 100 points total.

a. Write an equation relating the number of questions Jae got right to the total score he will get on the test.
b. If a score of 70 points earns a grade of $C-$, how many questions would Jae need to get right to get a $C-$ on the test?
c. If a score of 83 points earns a grade of B, how many questions would Jae need to get right to get a B on the test?
d. Suppose Jae got a score of 60% and then was allowed to retake the test. On the retake, he got all the questions right that he got right the first time, and also got half the questions right that he got wrong the first time. What is his new score?

2.2 Variable Expressions

Introduction - The Language of Algebra

No one likes doing the same problem over and over again—that's why mathematicians invented algebra. Algebra takes the basic principles of math and makes them more general, so we can solve a problem once and then use that solution to solve a group of similar problems.

In arithmetic, you've dealt with numbers and their arithmetical operations (such as $+, -, \times, \div$). In algebra, we use symbols called **variables** (which are usually letters, such as $x, y, a, b, c, \ldots$) to represent numbers and sometimes processes.

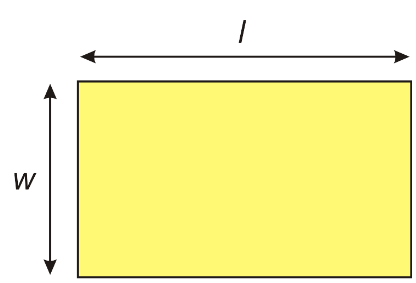
For example, we might use the letter x to represent some number we don't know yet, which we might need to figure out in the course of a problem. Or we might use two letters, like x and y, to show a relationship between two numbers without needing to know what the actual numbers are. The same letters can represent a wide range of possible numbers, and the same letter may represent completely different numbers when used in two different problems.

Using variables offers advantages over solving each problem "from scratch." With variables, we can:

- Formulate arithmetical laws such as $a+b=b+a$ for all real numbers a and b.
- Refer to "unknown" numbers. For instance: find a number x such that $3x+1=10$.
- Write more compactly about functional relationships such as, "If you sell x tickets, then your profit will be $3x-10$ dollars, or "$f(x)=3x-10$," where "f" is the profit function, and x is the input (i.e. how many tickets you sell).

Example 1

Write an algebraic expression for the perimeter and area of the rectangle below.



To find the perimeter, we add the lengths of all 4 sides. We can still do this even if we don't know the side lengths in numbers, because we can use variables like l and w to represent the unknown length and width. If we start at the top left and work clockwise, and if we use the letter P to represent the perimeter, then we can say:

$$P=l+w+l+w$$

We are adding 2 l's and 2 w's, so we can say that:

$$P=2\cdot l+2\cdot w$$

It's customary in algebra to omit multiplication symbols whenever possible. For example, $11x$ means the same thing as $11 \cdot x$ or $11 \times x$. We can therefore also write:

$$P = 2l + 2w$$

Area is *length multiplied by width*. In algebraic terms we get:

$$A = l \times w \rightarrow A = l \cdot w \rightarrow A = lw$$

Note: $2l + 2w$ by itself is an example of a **variable expression**; $P = 2l + 2w$ is an example of an **equation**. The main difference between expressions and **equa**tions is the presence of an **equa**ls sign (=).

In the above example, we found the simplest possible ways to express the perimeter and area of a rectangle when we don't yet know what its length and width actually are. Now, when we encounter a rectangle whose dimensions we do know, we can simply substitute (or **plug in**) those values in the above equations. In this chapter, we will encounter many expressions that we can evaluate by plugging in values for the variables involved.

Evaluate Algebraic Expressions

When we are given an algebraic expression, one of the most common things we might have to do with it is **evaluate** it for some given value of the variable. The following example illustrates this process.

Example 2

Let $x = 12$*. Find the value of* $2x - 7$.

To find the solution, we substitute 12 for x in the given expression. Every time we see x, we replace it with 12.

$$\begin{aligned} 2x - 7 &= 2(12) - 7 \\ &= 24 - 7 \\ &= 17 \end{aligned}$$

Note: At this stage of the problem, we place the substituted value in parentheses. We do this to make the written-out problem easier to follow, and to avoid mistakes. (If we didn't use parentheses and also forgot to add a multiplication sign, we would end up turning $2x$ into 212 instead of 2 times 12!)

Example 3

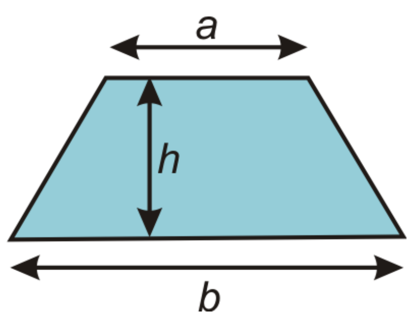
Let $y = -2$*. Find the value of* $\frac{7}{y} - 11y + 2$.

Solution

$$\begin{aligned} \frac{7}{(-2)} - 11(-2) + 2 &= -3\frac{1}{2} + 22 + 2 \\ &= 24 - 3\frac{1}{2} \\ &= 20\frac{1}{2} \end{aligned}$$

Many expressions have more than one variable in them. For example, the formula for the perimeter of a rectangle in the introduction has two variables: length (l) and width (w). In these cases, be careful to substitute the appropriate value in the appropriate place.

Example 4



The area of a trapezoid is given by the equation $A = \frac{h}{2}(a+b)$. *Find the area of a trapezoid with bases* $a = 10$ *cm and* $b = 15$ *cm and height* $h = 8$ *cm.*

To find the solution to this problem, we simply take the values given for the variables a, b, and h, and plug them in to the expression for A:

$$\begin{aligned}
A &= \frac{h}{2}(a+b) && \text{Substitute 10 for } a, \text{ 15 for } b, \text{ and 8 for } h. \\
A &= \frac{8}{2}(10+15) && \text{Evaluate piece by piece. } 10+15=25; \ \frac{8}{2}=4. \\
A &= 4(25) = 100
\end{aligned}$$

Solution: *The area of the trapezoid is 100 square centimeters.*

Evaluate Algebraic Expressions with Exponents

Many formulas and equations in mathematics contain exponents. Exponents are used as a short-hand notation for repeated multiplication. For example:

$$\begin{aligned}
2 \cdot 2 &= 2^2 \\
2 \cdot 2 \cdot 2 &= 2^3
\end{aligned}$$

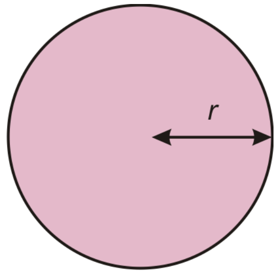
The exponent stands for how many times the number is used as a factor (multiplied). When we deal with integers, it is usually easiest to simplify the expression. We simplify:

$$\begin{aligned}
2^2 &= 4 \\
2^3 &= 8
\end{aligned}$$

However, we need exponents when we work with variables, because it is much easier to write x^8 than $x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$.

To evaluate expressions with exponents, substitute the values you are given for each variable and simplify. It is especially important in this case to substitute using parentheses in order to make sure that the simplification is done correctly.

Example 5



The area of a circle is given by the formula $A = \pi r^2$*. Find the area of a circle with radius* $r = 17$ *inches.*

Substitute values into the equation.

$$\begin{aligned} A &= \pi r^2 && \text{Substitute 17 for } r. \\ A &= \pi(17)^2 && \pi \cdot 17 \cdot 17 \approx 907.9202\ldots \text{ Round to 2 decimal places.} \end{aligned}$$

The area is approximately 907.92 square inches.

Example 6

Find the value of $\frac{x^2y^3}{x^3+y^2}$*, for* $x = 2$ *and* $y = -4$*.*

Substitute the values of x and y in the following.

$$\frac{x^2y^3}{x^3+y^2} = \frac{(2)^2(-4)^3}{(2)^3+(-4)^2} \qquad \text{Substitute 2 for } x \text{ and } -4 \text{ for } y.$$

$$\frac{4(-64)}{8+16} = \frac{-256}{24} = \frac{-32}{3} \qquad \text{Evaluate expressions: } (2)^2 = (2)(2) = 4 \text{ and}$$

$$(2)^3 = (2)(2)(2) = 8.\ (-4)^2 = (-4)(-4) = 16 \text{ and}$$

$$(-4)^3 = (-4)(-4)(-4) = -64.$$

Example 7

The height (h) *of a ball in flight is given by the formula* $h = -32t^2 + 60t + 20$*, where the height is given in feet and the time* (t) *is given in seconds. Find the height of the ball at time* $t = 2$ *seconds.*

Solution

$$\begin{aligned} h &= -32t^2 + 60t + 20 \\ &= -32(2)^2 + 60(2) + 20 && \text{Substitute 2 for } t. \\ &= -32(4) + 60(2) + 20 \\ &= 12 \end{aligned}$$

The height of the ball is 12 feet.

Review Questions

1. Write the following in a more condensed form by leaving out a multiplication symbol.
   a. $2 \times 11x$

b. $1.35 \cdot y$
c. $3 \times \frac{1}{4}$
d. $\frac{1}{4} \cdot z$

2. Evaluate the following expressions for $a = -3,\ b = 2,\ c = 5$, and $d = -4$.
   a. $2a + 3b$
   b. $4c + d$
   c. $5ac - 2b$
   d. $\frac{2a}{c-d}$
   e. $\frac{3b}{d}$
   f. $\frac{a-4b}{3c+2d}$
   g. $\frac{1}{a+b}$
   h. $\frac{ab}{cd}$
3. Evaluate the following expressions for $x = -1,\ y = 2,\ z = -3$, and $w = 4$.
   a. $8x^3$
   b. $\frac{5x^2}{6z^3}$
   c. $3z^2 - 5w^2$
   d. $x^2 - y^2$
   e. $\frac{z^3+w^3}{z^3-w^3}$
   f. $2x^3 - 3x^2 + 5x - 4$
   g. $4w^3 + 3w^2 - w + 2$
   h. $3 + \frac{1}{z^2}$
4. The weekly cost C of manufacturing x remote controls is given by the formula $C = 2000 + 3x$, where the cost is given in dollars.
   a. What is the cost of producing 1000 remote controls?
   b. What is the cost of producing 2000 remote controls?
   c. What is the cost of producing 2500 remote controls?
5. The volume of a box without a lid is given by the formula $V = 4x(10 - x)^2$, where x is a length in inches and V is the volume in cubic inches.
   a. What is the volume when $x = 2$?
   b. What is the volume when $x = 3$?

2.3 Equations and Inequalities

Introduction

In algebra, an **equation** is a mathematical expression that contains an equals sign. It tells us that two expressions represent the same number. For example, $y = 12x$ is an equation. An **inequality** is a mathematical expression that contains inequality signs. For example, $y \leq 12x$ is an inequality. Inequalities are used to tell us that an expression is either larger or smaller than another expression. Equations and inequalities can contain both **variables** and **constants**.

Variables are usually given a letter and they are used to represent unknown values. These quantities can change because they depend on other numbers in the problem.

Constants are quantities that remain unchanged. Ordinary numbers like 2, -3, $\frac{3}{4}$, and π are constants.

Equations and inequalities are used as a shorthand notation for situations that involve numerical data. They are very useful because most problems require several steps to arrive at a solution, and it becomes tedious to repeatedly write out the situation in words.

Write Equations and Inequalities

Here are some examples of equations:

$$3x-2=5 \qquad x+9=2x+5 \qquad \frac{x}{3}=15 \qquad x^2+1=10$$

To write an inequality, we use the following symbols:

>greater than

$\geq$ greater than or equal to

<less than

$\leq$ less than or equal to

$\neq$ not equal to

Here are some examples of inequalities:

$$3x<5 \qquad 4-x\leq 2x \qquad x^2+2x-1>0 \qquad \frac{3x}{4}\geq\frac{x}{2}-3$$

The most important skill in algebra is the ability to translate a word problem into the correct equation or inequality so you can find the solution easily. The first two steps are **defining the variables** and **translating** the word problem into a mathematical equation.

Defining the variables means that we assign letters to any unknown quantities in the problem.

Translating means that we change the word expression into a mathematical expression containing variables and mathematical operations with an equal sign or an inequality sign.

Example 1

Define the variables and translate the following expressions into equations.

a) A number plus 12 is 20.

b) 9 less than twice a number is 33.

c) $20 was one quarter of the money spent on the pizza.

Solution

a) **Define**

Let $n =$ the number we are seeking.

Translate

A number plus 12 is 20.

$$n+12=20$$

b) **Define**

Let $n =$ the number we are seeking.

Translate

9 less than twice a number is 33.

This means that twice the number, minus 9, is 33.

$$2n-9=33$$

c) **Define**

Let $m =$ the money spent on the pizza.

Translate

$20 was one quarter of the money spent on the pizza.

$$20=\frac{1}{4}m$$

Often word problems need to be reworded before you can write an equation.

Example 2

Find the solution to the following problems.

a) Shyam worked for two hours and packed 24 boxes. How much time did he spend on packing one box?

b) After a 20% discount, a book costs $12. How much was the book before the discount?

Solution

a) **Define**

Let $t =$ time it takes to pack one box.

Translate

Shyam worked for two hours and packed 24 boxes. This means that two hours is 24 times the time it takes to pack one box.

$$2 = 24t$$

Solve

$$t = \frac{2}{24} = \frac{1}{12} \text{ hours}$$
$$\frac{1}{12} \times 60 \text{ minutes} = 5 \text{ minutes}$$

Answer

Shyam takes 5 minutes to pack a box.

b) **Define**

Let $p =$ the price of the book before the discount.

Translate

After a 20% discount, the book costs $12. This means that the price minus 20% of the price is $12.

$$p - 0.20p = 12$$

Solve

$$p - 0.20p = 0.8p, \text{ so } 0.8p = 12$$
$$p = \frac{12}{0.8} = 15$$

Answer

The price of the book before the discount was $15.

Check

If the original price was $15, then the book was discounted by 20% of $15, or $3. $15 − 3 = $12. **The answer checks out**.

Example 3

Define the variables and translate the following expressions into inequalities.

a) The sum of 5 and a number is less than or equal to 2.

b) The distance from San Diego to Los Angeles is less than 150 miles.

c) Diego needs to earn more than an 82 on his test to receive a *B* in his algebra class.

d) A child needs to be 42 inches or more to go on the roller coaster.

Solution

a) **Define**

Let $n =$ the unknown number.

Translate

$$5 + n \leq 2$$

b) **Define**

Let $d =$ the distance from San Diego to Los Angeles in miles.

Translate

$$d < 150$$

c) **Define**

Let $x =$ Diego's test grade.

Translate

$$x > 82$$

d) **Define**

Let $h =$ the height of child in inches.

Translate:

$$h \geq 42$$

Check Solutions to Equations

You will often need to check solutions to equations in order to check your work. In a math class, checking that you arrived at the correct solution is very good practice. We check the solution to an equation by replacing the variable in an equation with the value of the solution. A solution should result in a true statement when plugged into the equation.

Example 4

Check that the given number is a solution to the corresponding equation.

a) $y = -1;\ 3y + 5 = -2y$

b) $z = 3;\ z^2 + 2z = 8$

c) $x = -\frac{1}{2};\ 3x + 1 = x$

Solution

Replace the variable in each equation with the given value.

a)

$$\begin{aligned} 3(-1) + 5 &= -2(-1) \\ -3 + 5 &= 2 \\ 2 &= 2 \end{aligned}$$

This is a true statement. This means that $y = -1$ is a solution to $3y+5 = -2y$.

b)

$$\begin{aligned} 3^2+2(3) &= 8 \\ 9+6 &= 8 \\ 15 &= 8 \end{aligned}$$

This is not a true statement. This means that $z = 3$ is **not a solution** to $z^2+2z = 8$.

c)

$$\begin{aligned} 3\left(-\frac{1}{2}\right)+1 &= -\frac{1}{2} \\ \left(-\frac{3}{2}\right)+1 &= -\frac{1}{2} \\ -\frac{1}{2} &= -\frac{1}{2} \end{aligned}$$

This is a true statement. This means that $x = -\frac{1}{2}$ is a solution to $3x+1 = x$.

Check Solutions to Inequalities

To check the solution to an inequality, we replace the variable in the inequality with the value of the solution. A solution to an inequality produces a true statement when substituted into the inequality.

Example 5

Check that the given number is a solution to the corresponding inequality.

a) $a = 10$; $20a \leq 250$

b) $b = -0.5$; $\frac{3-b}{b} > -4$

c) $x = \frac{3}{4}$; $4x+5 \leq 8$

Solution

Replace the variable in each inequality with the given value.

a)

$$\begin{aligned} 20(10) &\leq 250 \\ 200 &\leq 250 \end{aligned}$$

This statement is true. This means that $a = 10$ is a solution to the inequality $20a \leq 250$.

Note that $a = 10$ is not the only solution to this inequality. If we divide both sides of the inequality by 20, we can write it as $a \leq 12.5$. This means that any number less than or equal to 12.5 is also a solution to the inequality.

b)

$$\begin{aligned} \frac{3-(-0.5)}{(-0.5)} &> -4 \\ \frac{3+0.5}{-0.5} &> -4 \\ -\frac{3.5}{0.5} &> -4 \\ -7 &> -4 \end{aligned}$$

This statement is false. This means that $b = -0.5$ is not a solution to the inequality $\frac{3-b}{b} > -4$.

c)

$$\begin{aligned} 4\left(\frac{3}{4}\right)+5 &\geq 8 \\ 3+5 &\geq 8 \\ 8 &\geq 8 \end{aligned}$$

This statement is true. It is true because this inequality includes an equals sign; since 8 is equal to itself, it is also "greater than or equal to" itself. This means that $x = \frac{3}{4}$ is a solution to the inequality $4x+5 \leq 8$.

Solve Real-World Problems Using an Equation

Let's use what we have learned about defining variables, writing equations and writing inequalities to solve some real-world problems.

Example 6

Tomatoes cost $0.50 each and avocados cost $2.00 each. Anne buys six more tomatoes than avocados. Her total bill is $8. How many tomatoes and how many avocados did Anne buy?

Solution

Define

Let $a =$ the number of avocados Anne buys.

Translate

Anne buys six more tomatoes than avocados. This means that $a+6 =$ the number of tomatoes.

Tomatoes cost $0.50 each and avocados cost $2.00 each. Her total bill is $8. This means that .50 times the number of tomatoes plus 2 times the number of avocados equals 8.

$$\begin{aligned} 0.5(a+6)+2a &= 8 \\ 0.5a+0.5\cdot 6+2a &= 8 \\ 2.5a+3 &= 8 \\ 2.5a &= 5 \\ a &= 2 \end{aligned}$$

Remember that $a =$ the number of avocados, so Anne buys two avocados. The number of tomatoes is $a+6 = 2+6 = 8$.

Answer

Anne bought 2 avocados and 8 tomatoes.

Check

If Anne bought two avocados and eight tomatoes, the total cost is: $(2\times 2)+(8\times 0.5) = 4+4 = 8$. **The answer checks out.**

Example 7

To organize a picnic Peter needs at least two times as many hamburgers as hot dogs. He has 24 hot dogs. What is the possible number of hamburgers Peter has?

Solution

Define

Let $x =$ number of hamburgers

Translate

Peter needs at least two times as many hamburgers as hot dogs. He has 24 hot dogs.

This means that twice the number of hot dogs is less than or equal to the number of hamburgers.

$$2 \times 24 \leq x, \text{ or } 48 \leq x$$

Answer

Peter needs at least 48 hamburgers.

Check

48 hamburgers is twice the number of hot dogs. So more than 48 hamburgers is more than twice the number of hot dogs. **The answer checks out**.

Review Questions

1. Define the variables and translate the following expressions into equations.
   a. Peter's Lawn Mowing Service charges $10 per job and $0.20 per square yard. Peter earns $25 for a job.
   b. Renting the ice-skating rink for a birthday party costs $200 plus $4 per person. The rental costs $324 in total.
   c. Renting a car costs $55 per day plus $0.45 per mile. The cost of the rental is $100.
   d. Nadia gave Peter 4 more blocks than he already had. He already had 7 blocks.
2. Define the variables and translate the following expressions into inequalities.
   a. A bus can seat 65 passengers or fewer.
   b. The sum of two consecutive integers is less than 54.
   c. The product of a number and 3 is greater than 30.
   d. An amount of money is invested at 5% annual interest. The interest earned at the end of the year is greater than or equal to $250.
   e. You buy hamburgers at a fast food restaurant. A hamburger costs $0.49. You have at most $3 to spend. Write an inequality for the number of hamburgers you can buy.
   f. Mariel needs at least 7 extra credit points to improve her grade in English class. Additional book reports are worth 2 extra credit points each. Write an inequality for the number of book reports Mariel needs to do.
3. Check whether the given number is a solution to the corresponding equation.
   a. $a = -3;\ 4a+3 = -9$
   b. $x = \frac{4}{3};\ \frac{3}{4}x + \frac{1}{2} = \frac{3}{2}$
   c. $y = 2;\ 2.5y - 10.0 = -5.0$
   d. $z = -5;\ 2(5-2z) = 20 - 2(z-1)$
4. Check whether the given number is a solution to the corresponding inequality.
   a. $x = 12;\ 2(x+6) \leq 8x$
   b. $z = -9;\ 1.4z + 5.2 > 0.4z$

c. $y = 40;\ -\frac{5}{2}y + \frac{1}{2} < -18$
d. $t = 0.4;\ 80 \geq 10(3t + 2)$

5. The cost of a Ford Focus is 27% of the price of a Lexus GS 450h. If the price of the Ford is $15000, what is the price of the Lexus?
6. On your new job you can be paid in one of two ways. You can either be paid $1000 per month plus 6% commission of total sales or be paid $1200 per month plus 5% commission on sales over $2000. For what amount of sales is the first option better than the second option? Assume there are always sales over $2000.
7. A phone company offers a choice of three text-messaging plans. Plan A gives you unlimited text messages for $10 a month; Plan B gives you 60 text messages for $5 a month and then charges you $0.05 for each additional message; and Plan C has no monthly fee but charges you $0.10 per message.

   a. If m is the number of messages you send per month, write an expression for the monthly cost of each of the three plans.
   b. For what values of m is Plan A cheaper than Plan B?
   c. For what values of m is Plan A cheaper than Plan C?
   d. For what values of m is Plan B cheaper than Plan C?
   e. For what values of m is Plan A the cheapest of all? (Hint: for what values is A both cheaper than B and cheaper than C?)
   f. For what values of m is Plan B the cheapest of all? (Careful—for what values is B cheaper than A?)
   g. For what values of m is Plan C the cheapest of all?
   h. If you send 30 messages per month, which plan is cheapest?
   i. What is the cost of each of the three plans if you send 30 messages per month?

2.4 1-Step Equations

Introduction

Nadia is buying a new mp3 player. Peter watches her pay for the player with a $100 bill. She receives $22.00 in change, and from only this information, Peter works out how much the player cost. How much was the player?

In algebra, we can solve problems like this using an **equation**. An **equation** is an algebraic expression that involves an **equals** sign. If we use the letter x to represent the cost of the mp3 player, we can write the equation $x+22=100$. This tells us that the value of the player **plus** the value of the change received is **equal** to the $100 that Nadia paid.

Another way we could write the equation would be $x=100-22$. This tells us that the value of the player is **equal** to the total amount of money Nadia paid $(100-22)$. This equation is mathematically equivalent to the first one, but it is easier to solve.

In this chapter, we will learn how to solve for the variable in a one-variable linear equation. **Linear equations** are equations in which each term is either a constant, or a constant times a single variable (raised to the first power). The term linear comes from the word line, because the graph of a linear equation is always a line.

We'll start with simple problems like the one in the last example.

Solving Equations Using Addition and Subtraction

When we work with an algebraic equation, it's important to remember that the two sides have to stay equal for the equation to stay true. We can change the equation around however we want, but whatever we do to one side of the equation, we have to do to the other side. In the introduction above, for example, we could get from the first equation to the second equation by subtracting 22 from both sides:

$$\begin{aligned}
x+22&=100\\
x+22-22&=100-22\\
x&=100-22
\end{aligned}$$

Similarly, we can add numbers to each side of an equation to help solve for our unknown.

Example 1

Solve $x-3=9$.

Solution

To solve an equation for x, we need to **isolate** x—that is, we need to get it by itself on one side of the equals sign. Right now our x has a 3 subtracted from it. To reverse this, we'll add 3—but we must add 3 to **both sides.**

$$\begin{aligned}
x-3&=9\\
x-3+3&=9+3\\
x+0&=9+3\\
x&=12
\end{aligned}$$

Example 2

Solve $z-9.7=-1.026$

Solution

It doesn't matter what the variable is—the solving process is the same.

$$\begin{aligned} z-9.7 &= -1.026 \\ z-9.7+9.7 &= -1.026+9.7 \\ z &= 8.674 \end{aligned}$$

Make sure you understand the addition of decimals in this example!

Example 3

Solve $x+\frac{4}{7}=\frac{9}{5}$.

Solution

To isolate x, we need to subtract $\frac{4}{7}$ from both sides.

$$\begin{aligned} x+\frac{4}{7} &= \frac{9}{5} \\ x+\frac{4}{7}-\frac{4}{7} &= \frac{9}{5}-\frac{4}{7} \\ x &= \frac{9}{5}-\frac{4}{7} \end{aligned}$$

Now we have to subtract fractions, which means we need to find the LCD. Since 5 and 7 are both prime, their lowest common multiple is just their product, 35.

$$\begin{aligned} x &= \frac{9}{5}-\frac{4}{7} \\ x &= \frac{7\cdot 9}{7\cdot 5}-\frac{4\cdot 5}{7\cdot 5} \\ x &= \frac{63}{35}-\frac{20}{35} \\ x &= \frac{63-20}{35} \\ x &= \frac{43}{35} \end{aligned}$$

Make sure you're comfortable with decimals and fractions! To master algebra, you'll need to work with them frequently.

Solving Equations Using Multiplication and Division

Suppose you are selling pizza for \$1.50 a slice and you can get eight slices out of a single pizza. How much money do you get for a single pizza? It shouldn't take you long to figure out that you get $8 \times \$1.50 = \$12.00$. You solved this problem by multiplying. Here's how to do the same thing algebraically, using x to stand for the cost in dollars of the whole pizza.

Example 4

Solve $\frac{1}{8} \cdot x = 1.5$.

Our x is being multiplied by one-eighth. To cancel that out and get x by itself, we have to multiply by the reciprocal, 8. Don't forget to multiply **both sides** of the equation.

$$8\left(\frac{1}{8} \cdot x\right) = 8(1.5)$$
$$x = 12$$

Example 5

Solve $\frac{9x}{5} = 5$.

$\frac{9x}{5}$ is equivalent to $\frac{9}{5} \cdot x$, so to cancel out that $\frac{9}{5}$, we multiply by the reciprocal, $\frac{5}{9}$.

$$\frac{5}{9}\left(\frac{9x}{5}\right) = \frac{5}{9}(5)$$
$$x = \frac{25}{9}$$

Example 6

Solve $0.25x = 5.25$.

0.25 is the decimal equivalent of one fourth, so to cancel out the 0.25 factor we would multiply by 4.

$$4(0.25x) = 4(5.25)$$
$$x = 21$$

Solving by division is another way to isolate x. Suppose you buy five identical candy bars, and you are charged \$3.25. How much did each candy bar cost? You might just divide \$3.25 by 5, but let's see how this problem looks in algebra.

Example 7

Solve $5x = 3.25$.

To cancel the 5, we divide both sides by 5.

$$\frac{5x}{5} = \frac{3.25}{5}$$
$$x = 0.65$$

Example 8

Solve $7x = \frac{5}{11}$.

Divide both sides by 7.

$$x = \frac{5}{11.7}$$
$$x = \frac{5}{77}$$

Example 9

Solve $1.375x = 1.2$.

Divide by 1.375

$$x = \frac{1.2}{1.375}$$
$$x = 0.8\overline{72}$$

Notice the bar above the final two decimals; it means that those digits recur, or repeat. The full answer is 0.872727272727272....

Solve Real-World Problems Using Equations

Example 10

In the year 2017, Anne will be 45years old. In what year was Anne born?

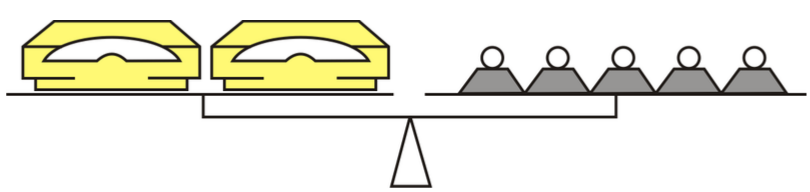
The unknown here is the year Anne was born, so that's our variable x. Here's our equation:

$$x + 45 = 2017$$
$$x + 45 - 45 = 2017 - 45$$
$$x = 1972$$

Anne was born in 1972.

Example 11

A mail order electronics company stocks a new mini DVD player and is using a balance to determine the shipping weight. Using only one-pound weights, the shipping department found that the following arrangement balances:



How much does each DVD player weigh?

Solution

Since the system balances, the total weight on each side must be equal. To write our equation, we'll use x for the weight of one DVD player, which is unknown. There are two DVD players, weighing a total of $2x$ pounds, on the left side of the balance, and on the right side are 5 1-pound weights, weighing a total of 5 pounds. So our equation is $2x = 5$. Dividing both sides by 2 gives us $x = 2.5$.

Each DVD player weighs 2.5 pounds.

Example 12

In 2004, Takeru Kobayashi of Nagano, Japan, ate 53.5 hot dogs in 12 minutes. This was 3 more hot dogs than his own previous world record, set in 2002. Calculate:

a) *How many minutes it took him to eat one hot dog.*

b) *How many hot dogs he ate per minute.*

c) *What his old record was.*

Solution

a) We know that the total time for 53.5 hot dogs is 12 minutes. We want to know the time for one hot dog, so that's x. Our equation is $53.5x = 12$. Then we divide both sides by 53.5 to get $x = \frac{12}{53.5}$, or $x = 0.224\ minutes$.

We can also multiply by 60 to get the time in seconds; 0.224 minutes is about 13.5 seconds. So that's how long it took Takeru to eat one hot dog.

b) Now we're looking for hot dogs per minute instead of minutes per hot dog. We'll use the variable y instead of x this time so we don't get the two confused. 12 minutes, times the number of hot dogs per minute, equals the total number of hot dogs, so $12y = 53.5$. Dividing both sides by 12 gives us $y = \frac{53.5}{12}$, or $y = 4.458$ hot dogs per minute.

c) We know that his new record is 53.5, and we know that's three more than his old record. If we call his old record z, we can write the following equation: $z+3 = 53.5$. Subtracting 3 from both sides gives us $z = 50.5$. So Takeru's old record was 50.5 hot dogs in 12 minutes.

Lesson Summary

- An equation in which each term is either a constant or the product of a constant and a single variable is a **linear equation**.
- We can add, subtract, multiply, or divide both sides of an equation by the same value and still have an equivalent equation.
- To solve an equation, **isolate** the unknown variable on one side of the equation by applying one or more arithmetic operations to both sides.

Review Questions

1. Solve the following equations for x.
   a. $x = 11 = 7$
   b. $x - 1.1 = 3.2$
   c. $7x = 21$
   d. $4x = 1$
   e. $\frac{5x}{12} = \frac{2}{3}$
   f. $x + \frac{5}{2} = \frac{2}{3}$
   g. $x - \frac{5}{6} = \frac{3}{8}$

h. $0.01x = 11$

2. Solve the following equations for the unknown variable.
   a. $q - 13 = -13$
   b. $z + 1.1 = 3.0001$
   c. $21s = 3$
   d. $t + \frac{1}{2} = \frac{1}{3}$
   e. $\frac{7f}{11} = \frac{7}{11}$
   f. $\frac{3}{4} = -\frac{1}{2} - y$
   g. $6r = \frac{3}{8}$
   h. $\frac{9b}{16} = \frac{3}{8}$
3. Peter is collecting tokens on breakfast cereal packets in order to get a model boat. In eight weeks he has collected 10 tokens. He needs 25 tokens for the boat. Write an equation and determine the following information.
   a. How many more tokens he needs to collect, n.
   b. How many tokens he collects per week, w.
   c. How many more weeks remain until he can send off for his boat, r.
4. Juan has baked a cake and wants to sell it in his bakery. He is going to cut it into 12 slices and sell them individually. He wants to sell it for three times the cost of making it. The ingredients cost him \$8.50, and he allowed \$1.25 to cover the cost of electricity to bake it. Write equations that describe the following statements
   a. The amount of money that he sells the cake for (u).
   b. The amount of money he charges for each slice (c).
   c. The total profit he makes on the cake (w).
5. Jane is baking cookies for a large party. She has a recipe that will make one batch of two dozen cookies, and she decides to make five batches. To make five batches, she finds that she will need 12.5 cups of flour and 15 eggs.
   a. How many cookies will she make in all?
   b. How many cups of flour go into one batch?
   c. How many eggs go into one batch?
   d. If Jane only has a dozen eggs on hand, how many more does she need to make five batches?
   e. If she doesn't go out to get more eggs, how many batches can she make? How many cookies will that be?

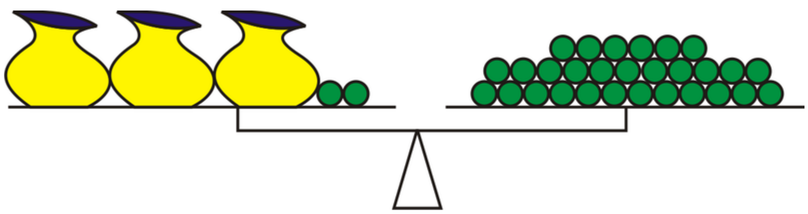
2.5 2-Step Equations

Solve a Two-Step Equation

We've seen how to solve for an unknown by isolating it on one side of an equation and then evaluating the other side. Now we'll see how to solve equations where the variable takes more than one step to isolate.

Example 1

Rebecca has three bags containing the same number of marbles, plus two marbles left over. She places them on one side of a balance. Chris, who has more marbles than Rebecca, adds marbles to the other side of the balance. He finds that with 29 marbles, the scales balance. How many marbles are in each bag? Assume the bags weigh nothing.



Solution

We know that the system balances, so the weights on each side must be equal. If we use x to represent the number of marbles in each bag, then we can see that on the left side of the scale we have three bags (each containing x marbles) plus two extra marbles, and on the right side of the scale we have 29 marbles. The balancing of the scales is similar to the balancing of the following equation.

$$3x+2=29$$

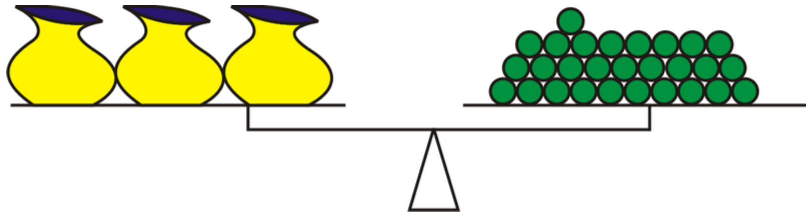
"Three bags plus two marbles **equals** 29 marbles"

To solve for x, we need to first get all the variables (terms containing an x) alone on one side of the equation. We've already got all the x's on one side; now we just need to isolate them.

$$\begin{aligned}
3x+2 &= 29 \\
3x+2-2 &= 29-2 && \text{Get rid of the 2 on the left by subtracting it from both sides.} \\
3x &= 27 \\
\frac{3x}{3} &= \frac{27}{3} && \text{Divide both sides by 3.} \\
x &= 9
\end{aligned}$$

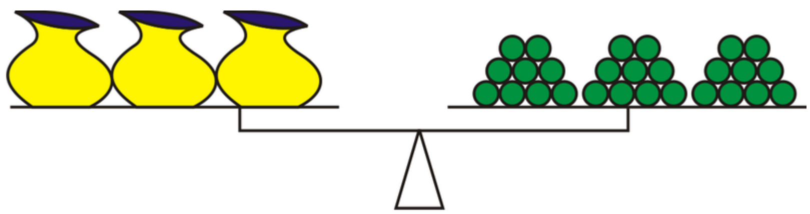
There are nine marbles in each bag.

We can do the same with the real objects as we did with the equation. Just as we subtracted 2 from both sides of the equals sign, we could remove two marbles from each side of the scale. Because we removed the same number of marbles from each side, we know the scales will still balance.



Then, because there are three bags of marbles on the left-hand side of the scale, we can divide the marbles on the right-hand side into three equal piles. You can see that there are nine marbles in each.

Three bags of marbles ***balances*** *three piles of nine marbles.*



So each bag of marbles balances nine marbles, meaning that each bag contains nine marbles.

Example 2

Solve $6(x+4)=12$.

This equation has the x buried in parentheses. To dig it out, we can proceed in one of two ways: we can either distribute the six on the left, or divide both sides by six to remove it from the left. Since the right-hand side of the equation is a multiple of six, it makes sense to divide. That gives us $x+4=2$. Then we can subtract 4 from both sides to get $x=-2$.

Example 3

Solve $\frac{x-3}{5}=7$.

It's always a good idea to get rid of fractions first. Multiplying both sides by 5 gives us $x-3=35$, and then we can add 3 to both sides to get $x=38$.

Example 4

Solve $\frac{5}{9}(x+1)=\frac{2}{7}$.

First, we'll cancel the fraction on the left by multiplying by the reciprocal (the multiplicative inverse).

$$\frac{9}{5}\cdot\frac{5}{9}(x+1)=\frac{9}{5}\cdot\frac{2}{7}$$
$$(x+1)=\frac{18}{35}$$

Then we subtract 1 from both sides. ($\frac{35}{35}$ is equivalent to 1.)

$$\begin{aligned}
x+1 &= \frac{18}{35} \\
x+1-1 &= \frac{18}{35}-\frac{35}{35} \\
x &= \frac{18-35}{35} \\
x &= \frac{-17}{35}
\end{aligned}$$

These examples are called **two-step equations**, because we need to perform two separate operations on the equation to isolate the variable.

Solve a Two-Step Equation by Combining Like Terms

When we look at a linear equation we see two kinds of terms: those that contain the unknown variable, and those that don't. When we look at an equation that has an x on both sides, we know that in order to solve it, we need to get all the $x-$terms on one side of the equation. This is called **combining like terms**. The terms with an x in them are **like terms** because they contain the same variable (or, as you will see in later chapters, the same combination of variables).

TABLE 2.8:

Like Terms	Unlike Terms
$4x, 10x, -3.5x$, and $\frac{x}{12}$	$3x$ and $3y$
$3y, 0.000001y$, and y	$4xy$ and $4x$
$xy, 6xy$, and $2.39xy$	$0.5x$ and 0.5

To add or subtract like terms, we can use the Distributive Property of Multiplication.

$$\begin{aligned}
3x+4x &= (3+4)x = 7x \\
0.03xy-0.01xy &= (0.03-0.01)xy = 0.02xy \\
-y+16y+5y &= (-1+16+5)y = 10y \\
5z+2z-7z &= (5+2-7)z = 0z = 0
\end{aligned}$$

To solve an equation with two or more like terms, we need to combine the terms first.

Example 5

Solve $(x+5)-(2x-3)=6$.

There are two like terms: the x and the $-2x$ (don't forget that the negative sign applies to everything in the parentheses). So we need to get those terms together. The associative and distributive properties let us rewrite the equation as $x+5-2x+3=6$, and then the commutative property lets us switch around the terms to get $x-2x+5+3=6$, or $(x-2x)+(5+3)=6$.

$(x-2x)$ is the same as $(1-2)x$, or $-x$, so our equation becomes $-x+8=6$

Subtracting 8 from both sides gives us $-x=-2$.

And finally, multiplying both sides by -1 gives us $x=2$.

Example 6

Solve $\frac{x}{2} - \frac{x}{3} = 6$.

This problem requires us to deal with fractions. We need to write all the terms on the left over a common denominator of six.

$$\frac{3x}{6} - \frac{2x}{6} = 6$$

Then we subtract the fractions to get $\frac{x}{6} = 6$.

Finally we multiply both sides by 6 to get $x = 36$.

Solve Real-World Problems Using Two-Step Equations

The hardest part of solving word problems is translating from words to an equation. First, you need to look to see what the equation is asking. What is the **unknown** for which you have to solve? That will be what your **variable** stands for. Then, follow what is going on with your variable all the way through the problem.

Example 7

An emergency plumber charges $65 as a call-out fee plus an additional $75 per hour. He arrives at a house at 9:30 and works to repair a water tank. If the total repair bill is $196.25, at what time was the repair completed?

In order to solve this problem, we collect the information from the text and convert it to an equation.

Unknown: time taken in hours –this will be our x

The bill is made up of two parts: a call out fee and a per-hour fee. The call out is a flat fee, and independent of x—it's the same no matter how many hours the plumber works. The per-hour part depends on the number of hours (x). So the total fee is $65 (no matter what) plus 75x$ (where x is the number of hours), or $65 + 75x$.

Looking at the problem again, we also can see that the total bill is $196.25. So our final equation is $196.25 = 65 + 75x$.

Solving for x:

$$\begin{aligned} 196.25 &= 65 + 75x && \text{Subtract 65 from both sides.} \\ 131.25 &= 75x && \text{Divide both sides by 75.} \\ 1.75 &= x && \text{The job took 1.75 hours.} \end{aligned}$$

Solution

The repair job was completed 1.75 hours after 9:30, so it was completed at 11:15AM.

Example 8

When Asia was young her Daddy marked her height on the door frame every month. Asia's Daddy noticed that between the ages of one and three, he could predict her height (in inches) by taking her age in months, adding 75 inches and multiplying the result by one-third. Use this information to answer the following:

a) *Write an equation linking her predicted height, h, with her age in months, m.*

b) *Determine her predicted height on her second birthday.*

c) *Determine at what age she is predicted to reach three feet tall.*

Solution

a) To convert the text to an equation, first determine the type of equation we have. We are going to have an equation that links **two variables**. Our unknown will change, depending on the information we are given. For example, we could solve for height given age, or solve for age given height. However, the text gives us a way to determine **height**. Our equation will start with "$h =$".

The text tells us that we can predict her height by taking her age in months, adding 75, and multiplying by $\frac{1}{3}$. So our equation is $h = (m+75) \cdot \frac{1}{3}$, or $h = \frac{1}{3}(m+75)$.

b) To predict Asia's height on her second birthday, we substitute $m = 24$ into our equation (because 2 years is 24 months) and solve for h.

$$\begin{aligned}
h &= \frac{1}{3}(24+75)\\
h &= \frac{1}{3}(99)\\
h &= 33
\end{aligned}$$

Asia's height on her second birthday was predicted to be 33 inches.

c) To determine the predicted age when she reached three feet, substitute $h = 36$ into the equation and solve for m.

$$\begin{aligned}
36 &= \frac{1}{3}(m+75)\\
108 &= m+75\\
33 &= m
\end{aligned}$$

Asia was predicted to be 33 months old when her height was three feet.

Example 9

To convert temperatures in Fahrenheit to temperatures in Celsius, follow the following steps: Take the temperature in degrees Fahrenheit and subtract 32. Then divide the result by 1.8 and this gives the temperature in degrees Celsius.

a) *Write an equation that shows the conversion process.*

b) *Convert 50 degrees Fahrenheit to degrees Celsius.*

c) *Convert 25 degrees Celsius to degrees Fahrenheit.*

d) *Convert -40 degrees Celsius to degrees Fahrenheit.*

a) The text gives the process to convert Fahrenheit to Celsius. We can write an equation using two variables. We will use f for temperature in Fahrenheit, and c for temperature in Celsius.

First we take the temperature in Fahrenheit and subtract 32.	$f-32$
Then divide by 1.8.	$\frac{f-32}{1.8}$
This equals the temperature in Celsius.	$c=\frac{f-32}{1.8}$

In order to convert from one temperature scale to another, simply substitute in for whichever temperature you know, and solve for the one you don't know.

b) To convert 50 degrees Fahrenheit to degrees Celsius, substitute $f=50$ into the equation.

$$c=\frac{50-32}{1.8}$$
$$c=\frac{18}{1.8}$$
$$c=10$$

50 degrees Fahrenheit is equal to 10 degrees Celsius.

c) To convert 25 degrees Celsius to degrees Fahrenheit, substitute $c=25$ into the equation:

$$25=\frac{f-32}{1.8}$$
$$45=f-32$$
$$77=f$$

25 degrees Celsius is equal to 77 degrees Fahrenheit.

d) To convert -40 degrees Celsius to degrees Fahrenheit, substitute $c=-40$ into the equation.

$$-40=\frac{f-32}{1.8}$$
$$-72=f-32$$
$$-40=f$$

-40 degrees Celsius is equal to -40 degrees Fahrenheit. (No, that's not a mistake! This is the one temperature where they are equal.)

Lesson Summary

- Some equations require more than one operation to solve. Generally it, is good to go from the outside in. If there are parentheses around an expression with a variable in it, cancel what is outside the parentheses first.
- Terms with the same variable in them (or no variable in them) are **like terms. Combine like terms** (adding or subtracting them from each other) to simplify the expression and solve for the unknown.

Review Questions

1. Solve the following equations for the unknown variable.
   a. $1.3x - 0.7x = 12$
   b. $6x - 1.3 = 3.2$
   c. $5x - (3x + 2) = 1$
   d. $4(x + 3) = 1$
   e. $5q - 7 = \frac{2}{3}$
   f. $\frac{3}{5}x + \frac{5}{2} = \frac{2}{3}$
   g. $s - \frac{3s}{8} = \frac{5}{6}$
   h. $0.1y + 11 = 0$
   i. $\frac{5q-7}{12} = \frac{2}{3}$
   j. $\frac{5(q-7)}{12} = \frac{2}{3}$
   k. $33t - 99 = 0$
   l. $5p - 2 = 32$
   m. $10y + 5 = 10$
   n. $10(y + 5) = 10$
   o. $10y + 5y = 10$
   p. $10(y + 5y) = 10$
2. Jade is stranded downtown with only \$10 to get home. Taxis cost \$0.75 per mile, but there is an additional \$2.35 hire charge. Write a formula and use it to calculate how many miles she can travel with her money.
3. Jasmin's Dad is planning a surprise birthday party for her. He will hire a bouncy castle, and will provide party food for all the guests. The bouncy castle costs \$150 for the afternoon, and the food will cost \$3 per person. Andrew, Jasmin's Dad, has a budget of \$300. Write an equation and use it to determine the maximum number of guests he can invite.
4. The local amusement park sells summer memberships for \$50 each. Normal admission to the park costs \$25; admission for members costs \$15.
   a. If Darren wants to spend no more than \$100 on trips to the amusement park this summer, how many visits can he make if he buys a membership with part of that money?
   b. How many visits can he make if he does not?
   c. If he increases his budget to \$160, how many visits can he make as a member?
   d. And how many as a non-member?
5. For an upcoming school field trip, there must be one adult supervisor for every five children.
   a. If the bus seats 40 people, how many children can go on the trip?
   b. How many children can go if a second 40-person bus is added?
   c. Four of the adult chaperones decide to arrive separately by car. Now how many children can go in the two buses?

2.6 Equations with Variables on Both Sides

Learning Objectives

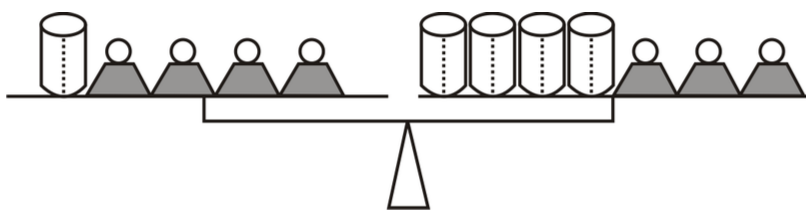
- Solve an equation with variables on both sides.
- Solve an equation with grouping symbols.
- Solve real-world problems using equations with variables on both sides.

Solve an Equation with Variables on Both Sides

When a variable appears on both sides of the equation, we need to manipulate the equation so that all variable terms appear on one side, and only constants are left on the other.

Example 1

Dwayne was told by his chemistry teacher to measure the weight of an empty beaker using a balance. Dwayne found only one lb weights, and so devised the following way of balancing the scales.



Knowing that each weight is one lb, calculate the weight of one beaker.

Solution

We know that the system balances, so the weights on each side must be equal. We can write an algebraic expression based on this fact. The unknown quantity, the weight of the beaker, will be our x. We can see that on the left hand scale we have one beaker and four weights. On the right scale, we have four beakers and three weights. The balancing of the scales is analogous to the balancing of the following equation:

$$x+4=4x+3$$

"One beaker plus 4 lbs **equals** 4 beakers plus 3 lbs"

To solve for the weight of the beaker, we want all the constants (numbers) on one side and all the variables (terms with x in them) on the other side. Since there are more beakers on the right and more weights on the left, we'll try to move all the x terms (beakers) to the right, and the constants (weights) to the left.

First we subtract 3 from both sides to get $x+1=4x$.

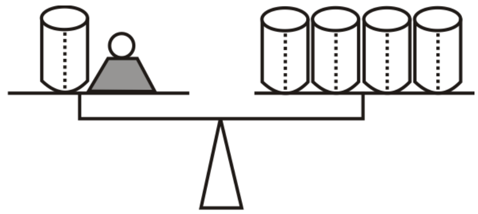
Then we subtract x from both sides to get $1=3x$.

Finally we divide by 3 to get $\frac{1}{3}=x$.

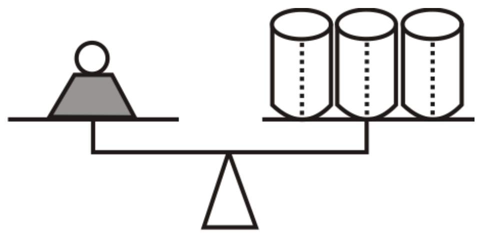
The weight of the beaker is one-third of a pound.

We can do the same with the real objects as we did with the equation. Just as we subtracted amounts from each side of the equation, we could remove a certain number of weights or beakers from each scale. Because we remove the same number of objects from each side, we know the scales will still balance.

First, we could remove three weights from each scale. This would leave one beaker and one weight on the left and four beakers on the right (in other words $x+1=4x$):



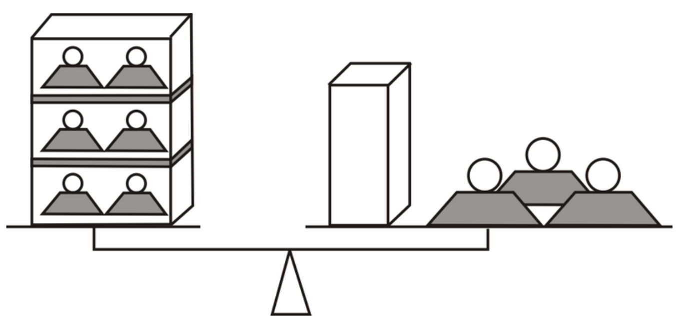
Then we could remove one beaker from each scale, leaving only one weight on the left and three beakers on the right, to get $1=3x$:



Looking at the balance, it is clear that the weight of one beaker is one-third of a pound.

Example 2

Sven was told to find the weight of an empty box with a balance. Sven found some one lb weights and five lb weights. He placed two one lb weights in three of the boxes and with a fourth empty box found the following way of balancing the scales:



Knowing that small weights are one lb and big weights are five lbs, calculate the weight of one box.

We know that the system balances, so the weights on each side must be equal. We can write an algebraic expression based on this equality. The unknown quantity—the weight of each empty box, in pounds—will be our x. A box with two 1 lb weights in it weighs $(x+2)$ pounds. Our equation, based on the picture, is $3(x+2)=x+3(5)$.

Distributing the 3 and simplifying, we get $3x+6=x+15$.

Subtracting x from both sides, we get $2x+6=15$.

Subtracting 6 from both sides, we get $2x=9$.

And finally we can divide by 2 to get $x=\frac{9}{2}$, or $x=4.5$.

Each box weighs 4.5 lbs.

Solve an Equation with Grouping Symbols

As you've seen, we can solve equations with variables on both sides even when some of the variables are in parentheses; we just have to get rid of the parentheses, and then we can start combining like terms. We use the same technique when dealing with fractions: first we multiply to get rid of the fractions, and then we can shuffle the terms around by adding and subtracting.

Example 3

Solve $3x+2 = \frac{5x}{3}$.

Solution

The first thing we'll do is get rid of the fraction. We can do this by multiplying both sides by 3, leaving $3(3x+2) = 5x$.

Then we distribute to get rid of the parentheses, leaving $9x+6 = 5x$.

We've already got all the constants on the left side, so we'll move the variables to the right side by subtracting $9x$ from both sides. That leaves us with $6 = -4x$.

And finally, we divide by -4 to get $-\frac{3}{2} = x$, or $x = -1.5$.

Example 4

Solve $7x+2 = \frac{5x-3}{6}$.

Solution

Again we start by eliminating the fraction. Multiplying both sides by 6 gives us $6(7x+2) = 5x-3$, and distributing gives us $42x+12 = 5x-3$.

Subtracting $5x$ from both sides gives us $37x+12 = -3$.

Subtracting 12 from both sides gives us $37x = -15$.

Finally, dividing by 37 gives us $x = -\frac{15}{37}$.

Example 5

Solve the following equation for x: $\frac{14x}{(x+3)} = 7$

Solution

The form of the left hand side of this equation is known as a **rational function** because it is the ratio of two other functions: $14x$ and $(x+3)$. But we can solve it just like any other equation involving fractions.

First we multiply both sides by $(x+3)$ to get rid of the fraction. Now our equation is $14x = 7(x+3)$.

Then we distribute: $14x = 7x+21$.

Then subtract $7x$ from both sides: $7x = 21$.

And divide by 7: $x = 3$.

Solve Real-World Problems Using Equations with Variables on Both Sides

Here's another chance to practice translating problems from words to equations. What is the equation asking? What is the **unknown** variable? What quantity will we use for our variable?

The text explains what's happening. Break it down into small, manageable chunks, and follow what's going on with our variable all the way through the problem.

More on Ohm's Law

Recall that the electrical current, I (amps), passing through an electronic component varies directly with the applied voltage, V (volts), according to the relationship $V = I \cdot R$ where R is the resistance measured in Ohms (Ω).

The resistance R of a number of components wired in a **series** (one after the other) is simply the sum of all the resistances of the individual components.

Example 6

In an attempt to find the resistance of a new component, a scientist tests it in series with standard resistors. A fixed voltage causes a 4.8 amp current in a circuit made up from the new component plus a 15Ω *resistor in series. When the component is placed in a series circuit with a* 50Ω *resistor, the same voltage causes a 2.0 amp current to flow. Calculate the resistance of the new component.*

This is a complex problem to translate, but once we convert the information into equations it's relatively straightforward to solve. First, we are trying to find the resistance of the new component (in Ohms, Ω). This is our x. We don't know the voltage that is being used, but we can leave that as a variable, V. Our first situation has a total resistance that equals the unknown resistance plus 15Ω. The current is 4.8 amps. Substituting into the formula $V = I \cdot R$, we get $V = 4.8(x+15)$.

Our second situation has a total resistance that equals the unknown resistance plus 50Ω. The current is 2.0 amps. Substituting into the same equation, this time we get $V = 2(x+50)$.

We know the voltage is fixed, so the V in the first equation must equal the V in the second. That means we can set the right-hand sides of the two equations equal to each other: $4.8(x+15) = 2(x+50)$. Then we can solve for x.

Distribute the constants first: $4.8x+72 = 2x+100$.

Subtract $2x$ from both sides: $2.8x+72 = 100$.

Subtract 72 from both sides: $2.8x = 28$.

Divide by 2.8: $x = 10$.

The resistance of the component is 10Ω**.**

Lesson Summary

If an unknown variable appears on both sides of an equation, distribute as necessary. Then simplify the equation to have the unknown on only one side.

Review Questions

1. Solve the following equations for the unknown variable.
   a. $3(x-1) = 2(x+3)$
   b. $7(x+20) = x+5$
   c. $9(x-2) = 3x+3$
   d. $2\left(a-\frac{1}{3}\right) = \frac{2}{5}\left(a+\frac{2}{3}\right)$
   e. $\frac{2}{7}\left(t+\frac{2}{3}\right) = \frac{1}{5}\left(t-\frac{2}{3}\right)$
   f. $\frac{1}{7}\left(v+\frac{1}{4}\right) = 2\left(\frac{3v}{2}-\frac{5}{2}\right)$
   g. $\frac{y-4}{11} = \frac{2}{5} \cdot \frac{2y+1}{3}$
   h. $\frac{z}{16} = \frac{2(3z+1)}{9}$
   i. $\frac{q}{16} + \frac{q}{6} = \frac{(3q+1)}{9} + \frac{3}{2}$
   j. $\frac{3}{x} = \frac{2}{x+1}$
   k. $\frac{5}{2+p} = \frac{3}{p-8}$
2. Manoj and Tamar are arguing about a number trick they heard. Tamar tells Andrew to think of a number, multiply it by five and subtract three from the result. Then Manoj tells Andrew to think of a number, add five and multiply the result by three. Andrew says that whichever way he does the trick he gets the same answer.
   a. What was the number Andrew started with?

b. What was the result Andrew got both times?
c. Name another set of steps that would have resulted in the same answer if Andrew started with the same number.

3. Manoj and Tamar try to come up with a harder trick. Manoj tells Andrew to think of a number, double it, add six, and then divide the result by two. Tamar tells Andrew to think of a number, add five, triple the result, subtract six, and then divide the result by three.
   a. Andrew tries the trick both ways and gets an answer of 10 each time. What number did he start out with?
   b. He tries again and gets 2 both times. What number did he start out with?
   c. Is there a number Andrew can start with that will *not* give him the same answer both ways?
   d. **Bonus:** Name another set of steps that would give Andrew the same answer every time as he would get from Manoj's and Tamar's steps.
4. I have enough money to buy five regular priced CDs and have $6 left over. However, all CDs are on sale today, for $4 less than usual. If I borrow $2, I can afford nine of them.
   a. How much are CDs on sale for today?
   b. How much would I have to borrow to afford nine of them if they weren't on sale?
5. Five identical electronics components were connected in series. A fixed but unknown voltage placed across them caused a 2.3 amp current to flow. When two of the components were replaced with standard 10Ω resistors, the current dropped to 1.9 amps. What is the resistance of each component?
6. Solve the following resistance problems. Assume the same voltage is applied to all circuits.
   a. Three unknown resistors plus 20Ω give the same current as one unknown resistor plus 70Ω.
   b. One unknown resistor gives a current of 1.5 amps and a 15Ω resistor gives a current of 3.0 amps.
   c. Seven unknown resistors plus 18Ω gives twice the current of two unknown resistors plus 150Ω.
   d. Three unknown resistors plus 1.5Ω gives a current of 3.6 amps and seven unknown resistors plus seven 12Ω resistors gives a current of 0.2 amps.

2.7 Multi-Step Equations

Learning Objectives

- Solve a multi-step equation by combining like terms.
- Solve a multi-step equation using the distributive property.
- Solve real-world problems using multi-step equations.

Solving Multi-Step Equations by Combining Like Terms

We've seen that when we solve for an unknown variable, it can take just one or two steps to get the terms in the right places. Now we'll look at solving equations that take several steps to isolate the unknown variable. Such equations are referred to as **multi-step equations**.

In this section, we'll simply be combining the steps we already know how to do. Our goal is to end up with all the constants on one side of the equation and all the variables on the other side. We'll do this by collecting like terms. Don't forget, like terms have the same combination of variables in them.

Example 1

Solve $\frac{3x+4}{3} - 5x = 6$.

Before we can combine the variable terms, we need to get rid of that fraction.

First let's put all the terms on the left over a common denominator of three: $\frac{3x+4}{3} - \frac{15x}{3} = 6$.

Combining the fractions then gives us $\frac{3x+4-15x}{3} = 6$.

Combining like terms in the numerator gives us $\frac{4-12x}{3} = 6$.

Multiplying both sides by 3 gives us $4 - 12x = 18$.

Subtracting 4 from both sides gives us $-12x = 14$.

And finally, dividing both sides by -12 gives us $x = -\frac{14}{12}$, which reduces to $x = -\frac{7}{6}$.

Solving Multi-Step Equations Using the Distributive Property

You may have noticed that when one side of the equation is multiplied by a constant term, we can either distribute it or just divide it out. If we can divide it out without getting awkward fractions as a result, then that's usually the better choice, because it gives us smaller numbers to work with. But if dividing would result in messy fractions, then it's usually better to distribute the constant and go from there.

Example 2

Solve $7(2x-5) = 21$.

The first thing we want to do here is get rid of the parentheses. We could use the Distributive Property, but it just so happens that 7 divides evenly into 21. That suggests that dividing both sides by 7 is the easiest way to solve this problem.

If we do that, we get $2x - 5 = \frac{21}{7}$ or just $2x - 5 = 3$. Then all we need to do is add 5 to both sides to get $2x = 8$, and then divide by 2 to get $x = 4$.

Example 3

Solve $17(3x+4) = 7$.

Once again, we want to get rid of those parentheses. We could divide both sides by 17, but that would give us an inconvenient fraction on the right-hand side. In this case, distributing is the easier way to go.

Distributing the 17 gives us $51x+68 = 7$. Then we subtract 68 from both sides to get $51x = -61$, and then we divide by 51 to get $x = \frac{-61}{51}$. (Yes, that's a messy fraction too, but since it's our final answer and we don't have to do anything else with it, we don't really care how messy it is.)

Example 4

Solve $4(3x-4)-7(2x+3) = 3$.

Before we can collect like terms, we need to get rid of the parentheses using the Distributive Property. That gives us $12x-16-14x-21 = 3$, which we can rewrite as $(12x-14x)+(-16-21) = 3$. This in turn simplifies to $-2x-37 = 3$.

Next we add 37 to both sides to get $-2x = 40$.

And finally, we divide both sides by -2 to get $x = -20$.

Example 5

Solve the following equation for x: $0.1(3.2+2x)+\frac{1}{2}\left(3-\frac{x}{5}\right) = 0$

This function contains both fractions and decimals. We should convert all terms to one or the other. It's often easier to convert decimals to fractions, but in this equation the fractions are easy to convert to decimals—and with decimals we don't need to find a common denominator!

In decimal form, our equation becomes $0.1(3.2+2x)+0.5(3-0.2x) = 0$.

Distributing to get rid of the parentheses, we get $0.32+0.2x+1.5-0.1x = 0$.

Collecting and combining like terms gives us $0.1x+1.82 = 0$.

Then we can subtract 1.82 from both sides to get $0.1x = -1.82$, and finally divide by 0.1 (or multiply by 10) to get $x = -18.2$.

Solving Real-World Problems Using Multi-Step Equations

Example 6

A growers' cooperative has a farmer's market in the town center every Saturday. They sell what they have grown and split the money into several categories. 8.5% of all the money taken in is set aside for sales tax. $150 goes to pay the rent on the space they occupy. What remains is split evenly between the seven growers. How much total money is taken in if each grower receives a $175 share?

Let's translate the text above into an equation. The unknown is going to be the total money taken in dollars. We'll call this x.

"8.5% of all the money taken in is set aside for sales tax." This means that 91.5% of the money remains. This is $0.915x$.

"$150 goes to pay the rent on the space they occupy." This means that what's left is $0.915x-150$.

"What remains is split evenly between the 7 growers." That means each grower gets $\frac{0.915x-150}{7}$.

If each grower's share is $175, then our equation to find x is $\frac{0.915x-150}{7} = 175$.

First we multiply both sides by 7 to get $0.915x-150 = 1225$.

Then add 150 to both sides to get $0.915x = 1375$.

Finally divide by 0.915 to get $x \approx 1502.7322$. Since we want our answer in dollars and cents, we round to two decimal places, or \$1502.73.

The workers take in a total of \$1502.73.

Example 7

A factory manager is packing engine components into wooden crates to be shipped on a small truck. The truck is designed to hold sixteen crates, and will safely carry a 1200 lb cargo. Each crate weighs 12 lbs empty. How much weight should the manager instruct the workers to put in each crate in order to get the shipment weight as close as possible to 1200 lbs?

The unknown quantity is the weight to put in each box, so we'll call that x.

Each crate when full will weigh $x+12$ *lbs*, so all 16 crates together will weigh $16(x+12)$ *lbs*.

We also know that all 16 crates together should weigh 1200 lbs, so we can say that $16(x+12) = 1200$.

To solve this equation, we can start by dividing both sides by 16: $x+12 = \frac{1200}{16} = 75$.

Then subtract 12 from both sides: $x = 63$.

The manager should tell the workers to put 63 lbs of components in each crate.

Ohm's Law

The electrical current, I (amps), passing through an electronic component varies directly with the applied voltage, V (volts), according to the relationship $V = I \cdot R$ where R is the resistance measured in Ohms (Ω).

Example 8

A scientist is trying to deduce the resistance of an unknown component. He labels the resistance of the unknown component x Ω*. The resistance of a circuit containing a number of these components is* $(5x+20)\Omega$*. If a 120 volt potential difference across the circuit produces a current of 2.5 amps, calculate the resistance of the unknown component.*

Solution

To solve this, we need to start with the equation $V = I \cdot R$ and substitute in $V = 120, I = 2.5$, and $R = 5x+20$. That gives us $120 = 2.5(5x+20)$.

Distribute the 2.5 to get $120 = 12.5x+50$.

Subtract 50 from both sides to get $70 = 12.5x$.

Finally, divide by 12.5 to get $5.6 = x$.

The unknown components have a resistance of $5.6\ \Omega$**.**

Distance, Speed and Time

The speed of a body is the distance it travels per unit of time. That means that we can also find out how far an object moves in a certain amount of time if we know its speed: we use the equation "distance = speed × time."

Example 8

Shanice's car is traveling 10 miles per hour slower than twice the speed of Brandon's car. She covers 93 miles in 1 hour 30 minutes. How fast is Brandon driving?

Solution

Here, we don't know either Brandon's speed or Shanice's, but since the question asks for Brandon's speed, that's what we'll use as our variable x.

The distance Shanice covers in miles is 93, and the time in hours is 1.5. Her speed is 10 less than twice Brandon's speed, or $2x-10$ miles per hour. Putting those numbers into the equation gives us $93 = 1.5(2x-10)$.

First we distribute, to get $93 = 3x-15$.

Then we add 15 to both sides to get $108 = 3x$.

Finally we divide by 3 to get $36 = x$.

Brandon is driving at 36 miles per hour.

We can check this answer by considering the situation another way: we can solve for Shanice's speed instead of Brandon's and then check that against Brandon's speed. We'll use y for Shanice's speed since we already used x for Brandon's.

The equation for Shanice's speed is simply $93 = 1.5y$. We can divide both sides by 1.5 to get $62 = y$, so Shanice is traveling at 62 miles per hour.

The problem tells us that Shanice is traveling 10 mph slower than twice Brandon's speed; that would mean that 62 is equal to 2 times 36 minus 10. Is that true? Well, 2 times 36 is 72, minus 10 is 62. The answer checks out.

In algebra, there's almost always more than one method of solving a problem. If time allows, it's always a good idea to try to solve the problem using two different methods just to confirm that you've got the answer right.

Speed of Sound

The speed of sound in dry air, v, is given by the equation $v = 331+0.6T$, where T is the temperature in Celsius and v is the speed of sound in meters per second.

Example 9

Tashi hits a drainpipe with a hammer and 250 meters away Minh hears the sound and hits his own drainpipe. Unfortunately, there is a one second delay between him hearing the sound and hitting his own pipe. Tashi accurately measures the time between her hitting the pipe and hearing Mihn's pipe at 2.46 seconds. What is the temperature of the air?

This is a complex problem and we need to be careful in writing our equations. First of all, the distance the sound travels is equal to the speed of sound multiplied by the time, and the speed is given by the equation above. So the distance equals $(331+0.6T)\times \text{time}$, and the time is $2.46-1$ (because for 1 second out of the 2.46 seconds measured, there was no sound actually traveling). We also know that the distance is 250×2 (because the sound traveled from Tashi to Minh and back again), so our equation is $250\times 2 = (331+0.6T)(2.46-1)$, which simplifies to $500 = 1.46(331+0.6T)$.

Distributing gives us $500 = 483.26+0.876T$, and subtracting 483.26 from both sides gives us $16.74 = 0.876T$. Then we divide by 0.876 to get $T \approx 19.1$.

The temperature is about 19.1 degrees Celsius.

Lesson Summary

- Multi-step equations are slightly more complex than one - and two-step equations, but use the same basic techniques.
- If dividing a number outside of parentheses will produce fractions, it is often better to use the **Distributive Property** to expand the terms and then combine like terms to solve the equation.

Review Questions

1. Solve the following equations for the unknown variable.

a. $3(x-1)-2(x+3)=0$
b. $3(x+3)-2(x-1)=0$
c. $7(w+20)-w=5$
d. $5(w+20)-10w=5$
e. $9(x-2)-3x=3$
f. $12(t-5)+5=0$
g. $2(2d+1)=\frac{2}{3}$
h. $2\left(5a-\frac{1}{3}\right)=\frac{2}{7}$
i. $\frac{2}{9}\left(i+\frac{2}{3}\right)=\frac{2}{5}$
j. $4\left(v+\frac{1}{4}\right)=\frac{35}{2}$
k. $\frac{g}{10}=\frac{6}{3}$
l. $\frac{s-4}{11}=\frac{2}{5}$
m. $\frac{2k}{7}=\frac{3}{8}$
n. $\frac{7x+4}{3}=\frac{9}{2}$
o. $\frac{9y-3}{6}=\frac{5}{2}$
p. $\frac{r}{3}+\frac{r}{2}=7$
q. $\frac{p}{16}-\frac{2p}{3}=\frac{1}{9}$
r. $\frac{m+3}{2}-\frac{m}{4}=\frac{1}{3}$
s. $5\left(\frac{k}{3}+2\right)=\frac{32}{3}$
t. $\frac{3}{z}=\frac{2}{5}$
u. $\frac{2}{r}+2=\frac{10}{3}$
v. $\frac{12}{5}=\frac{3+z}{z}$

2. An engineer is building a suspended platform to raise bags of cement. The platform has a mass of 200 kg, and each bag of cement is 40 kg. He is using two steel cables, each capable of holding 250 kg. Write an equation for the number of bags he can put on the platform at once, and solve it.
3. A scientist is testing a number of identical components of unknown resistance which he labels $x\Omega$. He connects a circuit with resistance $(3x+4)\Omega$ to a steady 12 volt supply and finds that this produces a current of 1.2 amps. What is the value of the unknown resistance?
4. Lydia inherited a sum of money. She split it into five equal parts. She invested three parts of the money in a high-interest bank account which added 10% to the value. She placed the rest of her inheritance plus $500 in the stock market but lost 20% on that money. If the two accounts end up with exactly the same amount of money in them, how much did she inherit?
5. Pang drove to his mother's house to drop off her new TV. He drove at 50 miles per hour there and back, and spent 10 minutes dropping off the TV. The entire journey took him 94 minutes. How far away does his mother live?

2.8 Rational Equations Using Proportions

Here you'll learn how to use proportions to find the solutions to rational equations.

Solution of Rational Equations

You are now ready to solve rational equations! There are two main methods you will learn to solve rational equations:

- Cross products
- Lowest common denominators

In this Concept you will learn how to solve using cross products.

Solving a Rational Proportion

When two rational expressions are equal, a **proportion** is created and can be solved using its cross products.

Example

For example, to solve $\frac{x}{5} = \frac{(x+1)}{2}$, cross multiply and the products are equal.

$$\frac{x}{5} = \frac{(x+1)}{2} \rightarrow 2(x) = 5(x+1)$$

Solve for x:

$$\begin{aligned}2(x) = 5(x+1) &\rightarrow 2x = 5x+5\\ 2x-5x &= 5x-5x+5\\ -3x &= 5\\ x &= -\frac{5}{3}\end{aligned}$$

Solve the following equations.

1. $\frac{2x+1}{4} = \frac{x-3}{10}$
2. $\frac{4x}{x+2} = \frac{5}{9}$
3. $\frac{5}{3x-4} = \frac{2}{x+1}$
4. $\frac{2}{x+3} - \frac{1}{x+4} = 0$

Mixed Review

7. Divide: $-2\frac{9}{10} \div -\frac{15}{8}$.
8. Solve for $g : -1.5\left(-3\frac{4}{5}+g\right)=\frac{201}{20}$.
9. Find the discriminant of $6x^2+3x+4=0$ and determine the nature of the roots.
10. Simplify $\frac{6b}{2b+2}+3$.
11. Simplify $\frac{8}{2x-4}-\frac{5x}{x-5}$.
12. Divide: $(7x^2+16x-10)\div(x+3)$.
13. Simplify $(n-1)*(3n+2)(n-4)$.

2.9 Solving Rational Equations using the LCD

Here you'll use the LCD of the expressions in a rational equation in order to solve for x.

A right triangle has leg lengths of $\frac{1}{2}$ and $\frac{1}{x}$ units. Its hypotenuse is 2 units. What is the value of x.

Guidance

In addition to using cross-multiplication to solve a rational equation, we can also use the LCD of all the rational expressions within the equation and eliminate the fraction. To demonstrate, we will walk through a few examples.

Example A

Solve $\frac{5}{2}+\frac{1}{x}=3$.

Solution: The LCD for 2 and x is $2x$. Multiply each term by $2x$, so that the denominators are eliminated. We will put the $2x$ over 1, when multiplying it by the fractions, so that it is easier to line up and cross-cancel.

$$\begin{aligned}\frac{5}{2}+\frac{1}{x}&=3\\ \frac{\cancel{2}x}{1}\cdot\frac{5}{\cancel{2}}+\frac{2\cancel{x}}{1}\cdot\frac{1}{\cancel{x}}&=2x\cdot 3\\ 5x+2&=6x\\ 2&=x\end{aligned}$$

Checking the answer, we have $\frac{5}{2}+\frac{1}{2}=3\rightarrow\frac{6}{2}=3$

Example B

Solve $\frac{5x}{x-2}=7+\frac{10}{x-2}$.

Solution: Because the denominators are the same, we need to multiply all three terms by $x-2$.

$$\begin{aligned}\frac{5x}{x-2}&=7+\frac{10}{x-2}\\ \cancel{(x-2)}\cdot\frac{5x}{\cancel{x-2}}&=(x-2)\cdot 7+\cancel{(x-2)}\cdot\frac{10}{\cancel{x-2}}\\ 5x&=7x-14+10\\ -2x&=-4\\ x&=2\end{aligned}$$

Checking our answer, we have: $\frac{5\cdot 2}{2-2}=7+\frac{10}{2-2}\rightarrow\frac{10}{0}=7+\frac{10}{0}$. Because the solution is the vertical asymptote of two of the expressions, $x=2$ is an extraneous solution. Therefore, there is no solution to this problem.

Example C

Solve $\frac{3}{x}+\frac{4}{5}=\frac{6}{x-2}$.

Solution: Determine the LCD for 5, x, and $x-2$. It would be the three numbers multiplied together: $5x(x-2)$. Multiply each term by the LCD.

$$\frac{3}{x}+\frac{4}{5}=\frac{6}{x-2}$$
$$\frac{5\cancel{x}(x-2)}{1}\cdot\frac{3}{\cancel{x}}+\frac{\cancel{5}x(x-2)}{1}\cdot\frac{4}{\cancel{5}}=\frac{5x\cancel{(x-2)}}{1}\cdot\frac{6}{\cancel{x-2}}$$
$$15(x-2)+4x(x-2)=30x$$

Multiplying each term by the entire LCD cancels out each denominator, so that we have an equation that we have learned how to solve in previous concepts. Distribute the 15 and $4x$, combine like terms and solve.

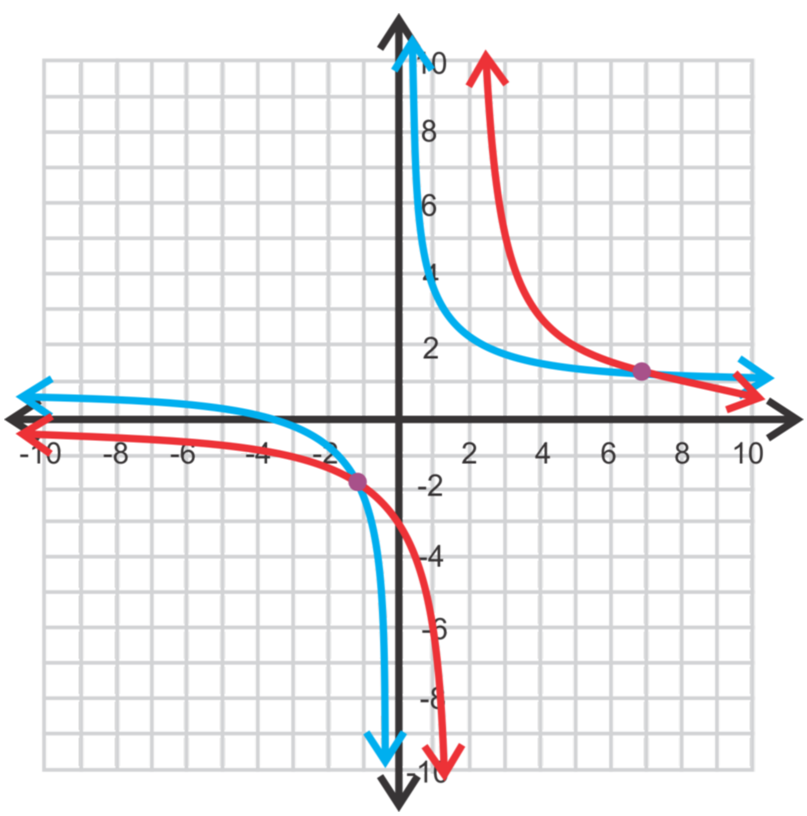
$$15x-30+4x^2-8x=30x$$
$$4x^2-23x-30=0$$

This polynomial is not factorable. Let's use the Quadratic Formula to find the solutions.

$x=\frac{23\pm\sqrt{(-23)^2-4\cdot4\cdot(-30)}}{2\cdot4}=\frac{23\pm\sqrt{1009}}{8}$

Approximately, the solutions are $\frac{23+\sqrt{1009}}{8}\approx 6.85$ and $\frac{23-\sqrt{1009}}{8}\approx -1.096$. It is harder to check these solutions. The easiest thing to do is to graph $\frac{3}{x}+\frac{4}{5}$ in $Y1$ and $\frac{6}{x-2}$ in $Y2$ (using your graphing calculator).

The x-values of the points of intersection (purple points in the graph) are approximately the same as the solutions we found.



Intro Problem Revisit We need to use the Pythagorean Theorem to solve for x.

$$\left(\frac{1}{2}\right)^2+\left(\frac{1}{x}\right)^2=2^2$$
$$\frac{1}{4}+\frac{1}{x^2}=4$$
$$\frac{\cancel{4}x^2}{1}\cdot\frac{1}{\cancel{4}}+\frac{4\cancel{x^2}}{1}\cdot\frac{1}{\cancel{x^2}}=4\cdot 4x^2$$
$$x^2+4=16x^2$$
$$4=15x^2$$
$$\frac{4}{15}=x^2$$
$$x=\frac{2\sqrt{15}}{15}$$

Guided Practice

Solve the following equations.

1. $\frac{2x}{x-3}=2+\frac{3x}{x^2-9}$
2. $\frac{4}{x-3}+5=\frac{9}{x+2}$
3. $\frac{3}{x^2+4x+4}+\frac{1}{x+2}=\frac{2}{x^2-4}$

Answers

1. The LCD is x^2-9. Multiply each term by its factored form to cross-cancel.

$$\frac{2x}{x-3}=2+\frac{3x}{x^2-9}$$
$$\frac{\cancel{(x-3)}(x+3)}{1}\cdot\frac{2x}{\cancel{x-3}}=(x-3)(x+3)\cdot 2+\frac{\cancel{(x-3)(x+3)}}{1}\cdot\frac{3x}{\cancel{x^2-9}}$$
$$2x(x+3)=2(x^2-9)+3x$$
$$2x^2+6x=2x^2-18+3x$$
$$3x=-18$$
$$x=-6$$

Checking our answer, we have: $\frac{2(-6)}{-6-3}=2+\frac{3(-6)}{(-6)^2-9}\rightarrow\frac{-12}{-9}=2+\frac{-18}{27}\rightarrow\frac{4}{3}=2-\frac{2}{3}$

2. The LCD is $(x-3)(x+2)$. Multiply each term by the LCD.

$$\frac{4}{x-3}+5=\frac{9}{x+2}$$
$$\cancel{(x-3)}(x+2)\cdot\frac{4}{\cancel{x-3}}+(x-3)(x+2)\cdot 5=(x-3)\cancel{(x+2)}\cdot\frac{9}{\cancel{x+2}}$$
$$4(x+2)+5(x-3)(x+2)=9(x-3)$$
$$4x+8+5x^2-5x-30=9x-27$$
$$5x^2-10x+5=0$$
$$5(x^2-2x+1)=0$$

This polynomial factors to be $5(x-1)(x-1)=0$, so $x=1$ is a repeated solution. Checking our answer, we have $\frac{4}{1-3}+5=\frac{9}{1+2}\rightarrow -2+5=3$

3. The LCD is $(x+2)(x+2)(x-2)$.

$$\frac{3}{x^2+4x+4}+\frac{1}{x+2}=\frac{2}{x^2-4}$$
$$\cancel{(x+2)(x+2)}(x-2)\cdot\frac{3}{\cancel{(x+2)(x+2)}}+\cancel{(x+2)}(x+2)(x-2)\cdot\frac{1}{\cancel{x+2}}=(x+2)\cancel{(x+2)(x-2)}\cdot\frac{2}{\cancel{(x-2)(x+2)}}$$
$$3(x-2)+(x-2)(x+2)=2(x+2)$$
$$3x-6+x^2-4=2x+4$$
$$x^2+x-14=0$$

This quadratic is not factorable, so we need to use the Quadratic Formula to solve for x.

$x=\frac{-1\pm\sqrt{1-4(-14)}}{2}=\frac{-1\pm\sqrt{57}}{2}\approx 3.27$ and -4.27

Using your graphing calculator, you can check the answer. The x-values of points of intersection of $y=\frac{3}{x^2+4x+4}+\frac{1}{x+2}$ and $y=\frac{2}{x^2-4}$ are the same as the values above.

Practice

Determine if the following values for x are solutions for the given equations.

1. $\frac{4}{x-3}+2=\frac{3}{x+4},\ x=-1$
2. $\frac{2x-1}{x-5}-3=\frac{x+6}{2x},\ x=6$

What is the LCD for each set of numbers?

3. $4-x,\ x^2-16$
4. $2x,\ 6x-12,\ x^2-9$
5. $x-3,\ x^2-x-6,\ x^2-4$

Solve the following equations.

6. $\frac{6}{x+2}+1=\frac{5}{x}$
7. $\frac{5}{3x}-\frac{2}{x+1}=\frac{4}{x}$

8. $\frac{12}{x^2-9} = \frac{8x}{x-3} - \frac{2}{x+3}$
9. $\frac{6x}{x^2-1} + \frac{2}{x+1} = \frac{3x}{x-1}$
10. $\frac{5x-3}{4x} - \frac{x+1}{x+2} = \frac{1}{x^2+2x}$
11. $\frac{4x}{x^2+6x+9} - \frac{2}{x+3} = \frac{3}{x^2-9}$
12. $\frac{x^2}{x^2-8x+16} = \frac{x}{x-4} + \frac{3x}{x^2-16}$
13. $\frac{5x}{2x-3} + \frac{x+1}{x} = \frac{6x^2+x+12}{2x^2-3x}$
14. $\frac{3x}{x^2+2x-8} = \frac{x+1}{x^2+4x} + \frac{2x+1}{x^2-2x}$
15. $\frac{x+1}{x^2+7x} + \frac{x+2}{x^2-3x} = \frac{x}{x^2+4x-21}$

2.10 Radical Equations

Here you'll learn how to find the solutions to radical equations.

Guidance

Solving radical equations is no different from solving linear or quadratic equations. Before you can begin to solve a radical equation, you must know how to cancel the radical. To do that, you must know its **inverse**.

TABLE 2.9:

Original Operation	Inverse Operation
Cube Root	Cubing (to the third power)
Square Root	Squaring (to the second power)
Fourth Root	Fourth power
"nth" Root	"nth" power

To solve a radical equation, you apply the solving equation steps you learned in previous Concepts, including the inverse operations for roots.

Example A

Solve $\sqrt{2x-1}=5$.

Solution:

The first operation that must be removed is the square root. Square both sides.

$$\begin{aligned}\left(\sqrt{2x-1}\right)^2 &= 5^2\\ 2x-1 &= 25\\ 2x &= 26\\ x &= 13\end{aligned}$$

Remember to check your answer by substituting it into the original problem to see if it makes sense.

Extraneous Solutions

Not every solution of a radical equation will check in the original problem. This is called an **extraneous solution**. This means you can find a solution using algebra, but it will not work when checked. This is because of the rule in a previous Concept:

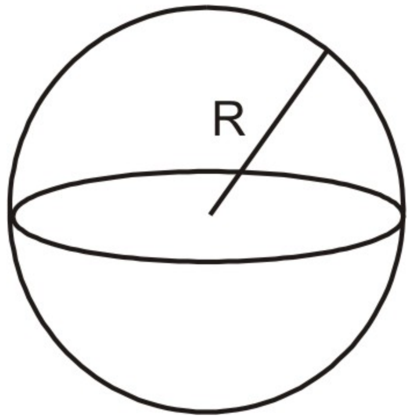
$\sqrt[n]{x}$ is undefined when n is an even whole number and $x < 0$,

or, in words, *even roots of negative numbers are undefined.*

Radical Equations in Real Life

Example B

A sphere has a volume of 456 cm^3. *If the radius of the sphere is increased by 2 cm, what is the new volume of the sphere?*



Solution:

1. **Define variables.** Let $R =$ the radius of the sphere.
2. **Find an equation.** The volume of a sphere is given by the formula: $V = \frac{4}{3}\pi r^3$.

By substituting 456 for the volume variable, the equation becomes $456 = \frac{4}{3}\pi r^3$.

$$\begin{aligned}
\text{Multiply by 3 :} &\quad 1368 = 4\pi r^3 \\
\text{Divide by } 4\pi : &\quad 108.92 = r^3 \\
\text{Take the cube root of each side:} &\quad r = \sqrt[3]{108.92} \Rightarrow r = 4.776\ cm \\
\text{The new radius is 2 centimeters more:} &\quad r = 6.776\ cm \\
\text{The new volume is :} &\quad V = \frac{4}{3}\pi(6.776)^3 = 1302.5\ cm^3
\end{aligned}$$

Check by substituting the values of the radii into the volume formula.
$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(4.776)^3 = 456\ cm^3$. The solution checks out.

Guided Practice

Solve $\sqrt{x+15} = \sqrt{3x-3}$.

Solution:

Begin by canceling the square roots by squaring both sides.

$$\begin{aligned}
\left(\sqrt{x+15}\right)^2 &= \left(\sqrt{3x-3}\right)^2 \\
x+15 &= 3x-3 \\
\text{Isolate the } x-\text{variable :} \quad 18 &= 2x \\
x &= 9
\end{aligned}$$

Check the solution: $\sqrt{9+15} = \sqrt{3(9)-3} \rightarrow \sqrt{24} = \sqrt{24}$. The solution checks.

Practice

In 1-16, find the solution to each of the following radical equations. Identify extraneous solutions.

1. $\sqrt{x+2}-2=0$
2. $\sqrt{3x-1}=5$
3. $2\sqrt{4-3x}+3=0$
4. $\sqrt[3]{x-3}=1$
5. $\sqrt[4]{x^2-9}=2$
6. $\sqrt[3]{-2-5x}+3=0$
7. $\sqrt{x^2-5x}-6=0$
8. $\sqrt{3x+4}=-6$
9. The area of a triangle is 24 in^2 and the height of the triangle is twice as long and the base. What are the base and the height of the triangle?
10. The volume of a square pyramid is given by the formula $V=\frac{A(h)}{3}$, where $A=$ *area of the base* and $h=$ *height of the pyramid*. The volume of a square pyramid is 1,600 cubic meters. If its height is 10 meters, find the area of its base.
11. The volume of a cylinder is 245 cm^3 and the height of the cylinder is one-third the diameter of the cylinder's base. The diameter of the cylinder is kept the same, but the height of the cylinder is increased by two centimeters. What is the volume of the new cylinder? $(\text{Volume} = \pi r^2 \cdot h)$
12. The height of a golf ball as it travels through the air is given by the equation $h=-16t^2+256$. Find the time when the ball is at a height of 120 feet.

Mixed Review

21. Joy sells two types of yarn: wool and synthetic. Wool is \$12 per skein and synthetic is \$9 per skein. If Joy sold 16 skeins of synthetic and collected a total of \$432, how many skeins of wool did she sell?
22. Solve $16 \geq |x-4|$.
23. Graph the solution: $\begin{cases} y \leq 2x-4 \\ y > -\frac{1}{4}x+6 \end{cases}$.
24. You randomly point to a day in the month of February, 2011. What is the probability your finger lands on a Monday?
25. Carbon-14 has a half life of 5,730 years. Your dog dug a bone from your yard. It had 93% of its carbon-14 remaining. How old is the bone?
26. What is true about solutions to inconsistent systems?

2.11 Solving Real-World Problems Using Multi-Step Equations

Here you'll learn how to translate words into to multi-step equations. You'll then solve such equations for their unknown variable.

Guidance

We can now use strategies for solving multi-step equations to solve real world equations.

Example A

A growers' cooperative has a farmer's market in the town center every Saturday. They sell what they have grown and split the money into several categories. 8.5% of all the money taken in is set aside for sales tax. $150 goes to pay the rent on the space they occupy. What remains is split evenly between the seven growers. How much total money is taken in if each grower receives a $175 share?

Let's translate the text above into an equation. The unknown is going to be the total money taken in dollars. We'll call this x.

"8.5% of all the money taken in is set aside for sales tax." This means that 91.5% of the money remains. This is $0.915x$.

"$150 goes to pay the rent on the space they occupy." This means that what's left is $0.915x - 150$.

"What remains is split evenly between the 7 growers." That means each grower gets $\frac{0.915x-150}{7}$.

If each grower's share is $175, then our equation to find x is $\frac{0.915x-150}{7} = 175$.

First we multiply both sides by 7 to get $0.915x - 150 = 1225$.

Then add 150 to both sides to get $0.915x = 1375$.

Finally divide by 0.915 to get $x \approx 1502.7322$. Since we want our answer in dollars and cents, we round to two decimal places, or $1502.73.

The workers take in a total of $1502.73.

Ohm's Law

The electrical current, I (amps), passing through an electronic component varies directly with the applied voltage, V (volts), according to the relationship $V = I \cdot R$ where R is the resistance measured in Ohms (Ω).

Example B

A scientist is trying to deduce the resistance of an unknown component. He labels the resistance of the unknown component x Ω. The resistance of a circuit containing a number of these components is $(5x+20)\Omega$. If a 120 volt potential difference across the circuit produces a current of 2.5 amps, calculate the resistance of the unknown component.

Solution

To solve this, we need to start with the equation $V = I \cdot R$ and substitute in $V = 120, I = 2.5$, and $R = 5x+20$. That gives us $120 = 2.5(5x+20)$.

Distribute the 2.5 to get $120 = 12.5x + 50$.

Subtract 50 from both sides to get $70 = 12.5x$.

Finally, divide by 12.5 to get $5.6 = x$.

The unknown components have a resistance of $5.6\ \Omega$**.**

Distance, Speed and Time

The speed of a body is the distance it travels per unit of time. That means that we can also find out how far an object moves in a certain amount of time if we know its speed: we use the equation "distance = speed × time."

Example C

Shanice's car is traveling 10 miles per hour slower than twice the speed of Brandon's car. She covers 93 miles in 1 hour 30 minutes. How fast is Brandon driving?

Solution

Here, we don't know either Brandon's speed or Shanice's, but since the question asks for Brandon's speed, that's what we'll use as our variable x.

The distance Shanice covers in miles is 93, and the time in hours is 1.5. Her speed is 10 less than twice Brandon's speed, or $2x - 10$ miles per hour. Putting those numbers into the equation gives us $93 = 1.5(2x - 10)$.

First we distribute, to get $93 = 3x - 15$.

Then we add 15 to both sides to get $108 = 3x$.

Finally we divide by 3 to get $36 = x$.

Brandon is driving at 36 miles per hour.

We can check this answer by considering the situation another way: we can solve for Shanice's speed instead of Brandon's and then check that against Brandon's speed. We'll use y for Shanice's speed since we already used x for Brandon's.

The equation for Shanice's speed is simply $93 = 1.5y$. We can divide both sides by 1.5 to get $62 = y$, so Shanice is traveling at 62 miles per hour.

The problem tells us that Shanice is traveling 10 mph slower than twice Brandon's speed; that would mean that 62 is equal to 2 times 36 minus 10. Is that true? Well, 2 times 36 is 72, minus 10 is 62. The answer checks out.

In algebra, there's almost always more than one method of solving a problem. If time allows, it's always a good idea to try to solve the problem using two different methods just to confirm that you've got the answer right.

Speed of Sound

The speed of sound in dry air, v, is given by the equation $v = 331 + 0.6T$, where T is the temperature in Celsius and v is the speed of sound in meters per second.

Example D

Tashi hits a drainpipe with a hammer and 250 meters away Minh hears the sound and hits his own drainpipe. Unfortunately, there is a one second delay between him hearing the sound and hitting his own pipe. Tashi accurately measures the time between her hitting the pipe and hearing Mihn's pipe at 2.46 seconds. What is the temperature of the air?

This is a complex problem and we need to be careful in writing our equations. First of all, the distance the sound travels is equal to the speed of sound multiplied by the time, and the speed is given by the equation above. So the distance equals $(331+0.6T)\times$ time, and the time is $2.46-1$ (because for 1 second out of the 2.46 seconds measured, there was no sound actually traveling). We also know that the distance is 250×2 (because the sound traveled from Tashi to Minh and back again), so our equation is $250\times 2=(331+0.6T)(2.46-1)$, which simplifies to $500=1.46(331+0.6T)$.

Distributing gives us $500=483.26+0.876T$, and subtracting 483.26 from both sides gives us $16.74=0.876T$. Then we divide by 0.876 to get $T\approx 19.1$.

The temperature is about 19.1 degrees Celsius.

Guided Practice

A factory manager is packing engine components into wooden crates to be shipped on a small truck. The truck is designed to hold sixteen crates, and will safely carry a 1200 lb cargo. Each crate weighs 12 lbs empty. How much weight should the manager instruct the workers to put in each crate in order to get the shipment weight as close as possible to 1200 lbs?

Solution:

The unknown quantity is the weight to put in each box, so we'll call that x.

Each crate when full will weigh $x+12$ *lbs*, so all 16 crates together will weigh $16(x+12)$ *lbs*.

We also know that all 16 crates together should weigh 1200 lbs, so we can say that $16(x+12)=1200$.

To solve this equation, we can start by dividing both sides by 16: $x+12=\frac{1200}{16}=75$.

Then subtract 12 from both sides: $x=63$.

The manager should tell the workers to put 63 lbs of components in each crate.

Practice

For 1-6, solve for the variable in the equation.

1. $\frac{s-4}{11}=\frac{2}{5}$
2. $\frac{2k}{7}=\frac{3}{8}$
3. $\frac{7x+4}{3}=\frac{9}{2}$
4. $\frac{9y-3}{6}=\frac{5}{2}$
5. $\frac{r}{3}+\frac{r}{2}=7$
6. $\frac{p}{16}-\frac{2p}{3}=\frac{1}{9}$
7. An engineer is building a suspended platform to raise bags of cement. The platform has a mass of 200 kg, and each bag of cement is 40 kg. He is using two steel cables, each capable of holding 250 kg. Write an equation for the number of bags he can put on the platform at once, and solve it.
8. A scientist is testing a number of identical components of unknown resistance which he labels $x\Omega$. He connects a circuit with resistance $(3x+4)\Omega$ to a steady 12 volt supply and finds that this produces a current of 1.2 amps. What is the value of the unknown resistance?
9. Lydia inherited a sum of money. She split it into five equal parts. She invested three parts of the money in a high-interest bank account which added 10% to the value. She placed the rest of her inheritance plus $500 in the stock market but lost 20% on that money. If the two accounts end up with exactly the same amount of money in them, how much did she inherit?
10. Pang drove to his mother's house to drop off her new TV. He drove at 50 miles per hour there and back, and spent 10 minutes dropping off the TV. The entire journey took him 94 minutes. How far away does his mother live?