Activity 2 (3.1, 3.2, 3.3) MTH 231
Week 2 Erickson

Name: _____

Instructions

Solve each of the following problems. You may write your answers on this activity or on a separate piece of paper. ***All relevant work should be shown for full credit.*** While completing the activity, you may discuss the problems with other students in this class, ask for help from the instructor or Lane tutors, or consult your class notes, the textbook for the class, and anything posted to our class Moodle site. Students may also use calculators such as Desmos. Students may not use online solution farms like Chegg or generative AI like ChatGPT.

1. (3 pts.) Find a *specific* counterexample to show that the following statement is false. Justify your answer.

$$\forall x, y \in \mathbb{R},\ \sqrt{x+y} = \sqrt{x} + \sqrt{y}\,.$$

2. (3 pts.) Which of the following is a negation for "All dogs are loyal"? More than one answer may be correct.
   a) All dogs are disloyal.
   b) No dogs are loyal.
   c) Some dogs are disloyal.
   d) Some dogs are loyal.
   e) There is a disloyal animal that is not a dog.
   f) There is a dog that is disloyal.
   g) No animals that are not dogs are loyal.
   h) Some animals that are not dogs are loyal.

3. (4 pts.) Write negations for each of the following statements and determine which is true, the given statement or its negation.

   a) $\exists x \in \mathbb{R}$ such that $x^2 = -1$.

   b) $\exists x \in \mathbb{R}$ such that $\forall y \in \mathbb{R}$, $xy = 0$.

   c) $\forall x \in \mathbb{R}$, $\exists y \in \mathbb{R}$ such that $x^2 + y^2 = 4$.